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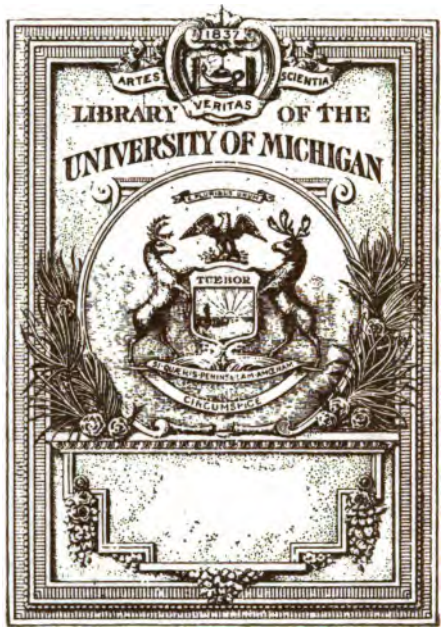

THE GIFT OF
Carnegie Inst of Technology

SECONDARY STRESSES IN BRIDGE TRUSSES

Introducing

The Graphic Method of Deformation Contour

and

Its Analytic Solution with Scientific Arrangement of Compntations

An Abstract of a THESIS Presented to the

Division of Science and Engineering

for the degree of

DOCTOR OF ENGINEERING

BY

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THIS PAPER IS

RESPECTFULLY DEDICATED TO

HENRY S. JACOBY

UNDER WHOSE INSTRUCTIONS

THE AUTHOR

FIRST ACQUIRED THE KNOWLEDGE OF

SECONDARY STRESSES

PREFACE

This paper is an abstract from a thesis entitled "Secondary Stresses in Framed Structures" presented by the writer to the Division of Science and Engineering, Carnegie Institute of Technology, as a partial requirement for the degree of Doctor of Engineering. In writing the original thesis three objects were kept in view: (1) to treat the subject of secondary stresses in a manner more logical, comprehensive and practical than is found elsewhere, (2) to introduce three new methods, two graphic and one analytic, for analyzing the secondary stresses, and (3) to investigate the effects of secondary stresses upon the design. In this paper two of the new methods are reproduced, with the materials entirely re-arranged and re-written. This is followed by a set of principles to be observed in reducing secondary stresses in bridge design.

The methods for computing secondary stresses have been greatly improved in recent years. Two objections, however, still remain: First, the amount of time involved is often excessive, and second, the lack of a systematic checking device by which the correctness of the various steps of procedure may be ensured. While there are numerous other defects, these two alone are generally sufficient to reduce their practical utility. Ever since the beginning of 1917 when the writer undertook the analysis of secondary stresses in a two-hinged arch, the results of which have been published in the Transactions of the American Society of Civil Engineers, vol. 82, p. 1104, it always occurred to him that there must be some method which is not only shorter and less cumbersome than the current ones but which also admits of a check. For two and a half years he has worked on the subject almost incessantly, striving to find some new method that will accomplish both the above mentioned results. At last, much to his satisfaction, the graphic method of deformation contour was obtained, which not only takes less time and checks itself, but also gives remarkably accurate results. Along with this method, almost contemporaneously, the analytic method was evolved. As these methods are new in their field

the writer has no hesitation to present them to the engineering world and it is his sincere hope that their usefulness be actually tested by further investigators.

In conclusion, the writer wishes to express his indebtedness to Professors H. R. Thayer and F. M. McCullough for suggestions and improvements which greatly enhance the value of the paper.

T. E. M.

Pittsburgh, Pa.,
December, 1919.

The above was written at a time when the abstract was to appear in the papers of an engineering society. On account of the author's returning home in China the arrangement was not carried out and the printing of the abstract was delayed. After corresponding with the Carnegie Institute of Technology for nearly a year the decision was finally reached to issue this paper in pamphlet form. This explanation is necessary because of some slight changes made in the original prefnce.

T. E. M.

Chinese Government Engineering College,
Tangshan, China,
December, 1920.

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INTRODUCTION

I. THE NATURE OF SECONDARY STRESS

In an articulated truss the members meeting at panel points may be connected either by pins or by stiff gusset plates. In the former arrangement the members are more or less free to turn about their joints and the angles between them may vary with the deformation of the truss. In the latter case, however, this is impossible, as the relative angles between the members must remain constant. This "Rigidity of joints" is the principal cause of the so-called secondary stresses.

Let a bridge truss with rigid joints be loaded in any manner. The various members will then change in length and the panel points will deflect. As the truss is made of triangles and a triangle cannot retain its shape after the sides have changed in length, the angles made by the members at the joints must also vary to accommodate the change. But the members cannot conform to this angle because the joints are rigid. The triangles must, therefore, lose their original forms and the sides must bend, as a result of the rotation of joints. Bending moments are thus produced in the members, reaching a maximum near the joints.* The fiber stresses resulting therefrom constitute the secondary stresses due to rigidity of joints.

II. THE TWO FUNDAMENTAL EQUATIONS

To analyze secondary stresses, certain conditions must be known that will give equations sufficient for the total unknowns. As a bridge is a static structure, one condition must be derived from the equilibrium of forces. It being also an elastic structure, another condition may be obtained from the elastic properties of the material of which the bridge is built. These two conditions give rise to two fundamental equations which, as will be seen later, are sufficient for solving the total unknowns.

*Taking the bending of the members into account, the maximum moments in compression members do not occur at the ends but somewhere between the ends.

1. The Equilibrium Equation

Since the truss is always in static equilibrium the resulting moment about any panel joint must be zero.* Hence, if the members 12, 13, 14, etc. of a truss meet at joint 1, and if M_{12}, M_{13}, M_{14}, etc. are the moments of the respective members about joint 1, then,

$$M_{12} + M_{13} + M_{14} + \ldots = 0,$$

Or, in general, ΣM about any joint $= 0$.

Let f = secondary stress produced by M,

I = moment of inertia of the member,

y = extreme fiber distance from the neutral axis for the same member,

S = section modulus of the member $= I/y$,

Then, $M = f I/y = f S$, and the above equation reduces to

$$\Sigma f S \text{ around any joint} = 0, \qquad (1)$$

which is the first fundamental equation known as the "equilibrium equation."

2. The Deformation Equation

As noted before, in a truss with rigid joints the deformations in members can not be taken care of by the change of angles, which the members make at the joints. On the other hand, these angles remain constant, and provision for deformations is made in some other way. Since the members are of elastic material, the simplest and easiest way would be to bend them; but they cannot bend without being subject to some outside influence and this outside influence is what produces the internal stress. In other words, some work must be performed *on* the member to produce this bending and as a consequence some corresponding internal work must be set up. Take, for illustration (Fig. 1) a beam, 12, subject to the influence of moment M which increases from 0 at 1 to M_2 at 2. (This is equivalent to a force equal to M_2/L placed at end 1, with end 2 fixed). The effect of M_2 on the beam is to rotate end 2 through an angle T_{21}, and end 1, through an angle T_{12}. These

*Eccentric moment considered as external.

rotations are brought about at the expense of an external work equal in amount to $1/2\ M_2\ T_{21}$. This induces an internal work in the beam equal to

$$\frac{1}{2}\int_0^L \frac{M_x^2\,\delta x}{EI} = \frac{1}{2}\,\frac{1}{EI}\int_0^L (M_2\frac{x}{L})^2\,\delta x = \frac{1}{2}\,\frac{M_2^2\,L}{3\,E\,I} = \frac{1}{2}M_2\ T_{21}$$

Or,
$$T_{21} = \frac{M_2\ L}{3\,EI} \qquad (a)$$

That is, the end 2 of beam 12 rotates through an angle T_{21} at the expense of an external moment M_2.

Similarly, the bending in truss members due to rigidity of joints is also brought about at the expense of external moments which become internal moments when the joint is considered as a whole. This bending may be considered as the result of two different rotations: the first due to the deformation of truss members and the other due to the rigidity of joints. Let 12, 13, 14, and 15 be four members meeting at joint 1, which is rigid (Fig. 2). After loading, on account of the deformations in members, the axis lines 12, 13, 14, etc., would have been displaced to 12′, 13′, etc. had the joint been a frictionless pin. The angles, $212' = H_{12}$, $313' = H_{13}$, etc., thus formed, are functions of the deformations and are different for different members. They could not, therefore, be actually realized in a truss with rigid joints, where a member is prevented from rotating relatively to the others. On the other hand, all the members must rotate through the same angle and the total rotations of members 12, 13, 14, etc., are not H_{12}, H_{13}, etc., but $B_1 - H_{12}$, $B_1 - H_{13}$, etc., where B_1 is the rotation of joint 1—the same for all members. This same result may also be obtained by considering: first, the axes 12′, 13′, etc., brought back to their respective original positions 12, 13, etc., and then, simultaneously, all the axes turned through a common angle, B_1.

These rotations, as mentioned before, must be produced at the expense of moments, and the work thus performed *on* members 12, 13, etc., is:

$$W_{12}=\frac{1}{2}M_{12}\,(B_1-H_{12}),$$

$$W_{13}=\frac{1}{2}M_{13}\,(B_1-H_{13}),\text{ etc.}$$

Since B_1 is the same for all the members at joint 1,

$$\frac{W_{12}}{M_{12}}-\frac{W_{13}}{M_{13}}=\frac{1}{2}\,(H_{13}-H_{12}),$$

$$\frac{W_{13}}{M_{13}}-\frac{W_{14}}{M_{14}}=\frac{1}{2}\,(H_{14}-H_{13}),\text{ etc.}\qquad(b)$$

Now, $H_{12}-H_{13}=$ angles $213-2'13'=$ change of angle between members 12 and 13 as a result of the loading. This change of angle is a function of the primary stresses in the members and also the proportions of the truss. If P_{12}, P_{13}, and P_{23} are the *unit* primary stresses in sides 12, 13, and 23 of triangle 123 and δA_{213}, δA_{132}, and δA_{321} are the changes in angles 213, 132, and 321 of the triangle it can be found by calculus that

$$E\delta A_{213}=(P_{23}-P_{13})\cot 132+(P_{23}-P_{12})\cot 123,$$

$$E\delta A_{132}=(P_{12}-P_{23})\cot 123+(P_{12}-P_{13})\cot 213,$$

$$E\delta A_{321}=(P_{13}-P_{12})\cot 213+(P_{13}-P_{23})\cot 132.\qquad(2)$$

As the terms in equation (2) are all known, the change of angles could be easily computed for the truss and $H_{12}-H_{13}=\delta A_{213}$, $H_{13}-H_{14}=\delta A_{314}$, etc., are therefore known. Substitute these values of δA for H in equation (b):

$$\frac{W_{12}}{M_{12}}-\frac{W_{13}}{M_{13}}=\frac{1}{2}\,\delta A_{213},$$

$$\frac{W_{13}}{M_{13}}-\frac{W_{14}}{M_{14}}=\frac{1}{2}\,\delta A_{314},\text{ etc.}\qquad(c)$$

Since the expression for internal work includes the total work done on the member, not only by M_{12} at end 1 of member 12 but also by M_{21} at end 2, the total external work on the member is

$$W = W_{12} + W_{21} = \frac{1}{2}(M_{12}T_{12} + M_{21}T_{21}),$$

and the internal work is

$$W = \frac{1}{2}\int_0 \frac{M_x^2\,\delta x}{EI},$$

where (Fig. 3) $M_x = M_a - M_b = M_{12} - \frac{x}{L}(M_{12} + M_{21})$.

Substitute this value of M_x in the above equation and integrate:

$$W = W_{12} + W_{21} = \frac{1}{2}\left\{\frac{1}{EI}\left(\frac{M_{12}^2 L}{3} + \frac{M_{21}^2 L}{3} - \frac{M_{12}M_{21}L}{3}\right)\right\} \quad (d)$$

It will be seen in equation (*d*) that the third and fourth terms represent work done on ends 1 and 2 of beam 12 by moments M_{12} and M_{21} respectively—by equation (*a*)—and the fifth term is the sum of works done by M_{12} on end 2 and M_{21} on end 1. But by Maxwell's theorem of reciprocal deflections the work done by M_{21} on end 1 and that done by M_{12} on end 2 are equal and, therefore, the difference of W_{12} and W_{21} is simply the difference of the first two terms in the above equation. Or,

$$W_{12} - W_{21} = \frac{1}{2}\left\{\frac{1}{EI}\left(\frac{M_{12}^2 L}{3} - \frac{M_{21}^2 L}{3}\right)\right\}. \quad (e)$$

Solving simultaneously equations (*d*) and (*e*),

$$W_{12} = \frac{1}{2}\frac{M_{12}L}{6EI}(2M_{12} - M_{21}).$$

Similarly, $$W_{13} = \frac{1}{2}\frac{M_{13}L}{6EI}(2M_{13} - M_{31}).$$

Substituting these values of W in equation (*c*), there is obtained

$$\frac{L_{12}}{6EI_{12}}(2M_{12} - M_{21}) - \frac{L_{13}}{6EI_{13}}(2M_{13} - M_{31}) = \delta A_{213}.$$

$$f_{pm} = \frac{1}{3}\left\{\frac{U_{lm}}{U_{mp}}(2\,f_{ml} - f_{lm}) + \frac{K_{lmp}}{U_{pm}}\right\} + \frac{2}{3}\left\{\frac{U_{lp}}{U_{pm}}(2\,f_{pl} - f_{lp}) - \frac{K_{lpm}}{U_{pm}}\right\}. \quad (8)$$

As a means of abbreviation, a new notation, r, will now be introduced to replace the difference of f, as defined by the following equation:

$$\begin{aligned} r_{lm} &= 2\,f_{lm} - f_{ml}, \\ r_{ml} &= 2\,f_{ml} - f_{lm}. \end{aligned} \quad (9)$$

On account of its very frequent occurrence in later discussions, the value of r_{lm} for member lm at l will be known as the "deformation modulus" of member lm at l.

Substitute r in the above equations:

$$r_{lp} = \frac{U_{lm}}{U_{lp}}\,r_{lm} + \frac{K_{plm}}{U_{lp}}, \quad (10)$$

$$f_{pl} = 2\,f_{lp} - \frac{U_{lm}}{U_{lp}}\,r_{lm} - \frac{K_{plm}}{U_{lp}}, \quad (11)$$

$$\left.\begin{aligned} f_{mp} &= \frac{2}{3}\left\{\frac{U_{lm}}{U_{pm}}r_{ml} + \frac{K_{lmp}}{U_{pm}}\right\} + \frac{1}{3}\left\{\frac{U_{lp}}{U_{pm}}r_{pl} - \frac{K_{lpm}}{U_{pm}}\right\} \\ f_{pm} &= \frac{1}{3}\left\{\frac{U_{lm}}{U_{pm}}r_{ml} + \frac{K_{lmp}}{U_{pm}}\right\} + \frac{2}{3}\left\{\frac{U_{lp}}{U_{pm}}r_{pl} - \frac{K_{lpm}}{U_{pm}}\right\} \end{aligned}\right\} \quad (12)$$

Solving equation (9) for f,

$$\begin{aligned} f_{lm} &= \frac{1}{3}(2\,r_{lm} + r_{ml}), \\ f_{ml} &= \frac{1}{3}(2\,r_{ml} + r_{lm}). \end{aligned} \quad (13)$$

From equation (13),

$$r_{ml} = 3\,f_{lm} - 2\,r_{lm}. \quad (14)$$

The above equations, which form the basis of the methods developed in this paper, are applied graphically to the "Method of Deformation Contour" in the following sections.

GRAPHIC ANALYSIS OF SECONDARY STRESSES BY THE METHOD OF DEFORMATION CONTOUR

IV. FUNDAMENTAL CONCEPTIONS

It has been shown that when a riveted truss is deformed, the members constituting the truss will be bent and bending moments developed along their axes. The moment in any member varies from end to end in a linear equation* and may be represented as such by a straight line. Since $M=f\,S$, and S is a constant for the member, the variation of f can also be represented by a straight line. More directly—*if S be considered as a stress acting in the member, and M the moment it produces about any point, then the offset of the stress from the point gives the value of f.*

Let 12 (Fig. 8) be a member connecting joints 1 and 2, and S its section modulus. If S be assumed as a compressive stress acting in this member, like the primary stress, it will produce moments at 1 and 2 if its line of action is displaced from axis 12, and the stresses produced at 1 and 2 will then be represented by the offsets f_{12} and f_{21}. Particularly, if the moments about 1 and 2 are those due to rigidity of joints, considering member 12 as a part of a riveted truss, the offsets f_{12} and f_{21} will be the secondary stresses.

The line of action of S will be known as the "secondary stress line," meaning that the secondary stress is given by the offset to this line from the axis of the member.†

Let the above conception be extended to every member of the truss. There will then be as many secondary stress lines as there are members. Each line will be displaced to an extent proportional to the bending moments produced. If, therefore, the lines are so located as to

*Neglecting the moment of primary stress due to deflection of the member from its straight axis.

†For the fiber where the secondary stress has the same sign as the moment.

satisfy the various imposed conditions as a result of the rigidity of joints the offsets of the secondary stress lines will give directly the secondary stresses. This conception is fundamental.

The first condition that the secondary stress lines are subject to is that the total resulting moment around any joint must be zero. Graphically, this means that if a force diagram be drawn of the secondary stress lines around any joint and an equilibrium polygon be likewise constructed for the joint, these two figures must respectively close. This, however, is impossible for the convention of secondary stress lines adopted above because the values of S, being constant, cannot be made to balance each other. Thus, in Fig. 9, stresses S_{12} and S_{13} cannot balance each other unless they are on the same straight line—a condition which is impossible. To overcome this difficulty, there will be introduced an external ideal force, R, with such a magnitude and direction that equilibrium is maintained around the joint. In other words, this force must be equal to the resultant of the secondary stress lines acting on the joint but with opposite sense in direction. Since there can be no external moment around the joint, its line of action must pass through the joint in consideration. Thus in Fig. 9 a force R must be introduced to balance the stresses S_{12} and S_{13}.

With the conceptions of S and R thus established it is now possible to draw a force diagram around every joint of the truss and to construct from this an equilibrium polygon. If these two polygons are made to close, this everywhere satisfies the first condition that the sum of internal moments around any joint must be zero. Further, from the condition that the total external moment on the truss is also zero, it may be inferred that both the force and equilibrium polygons of the external ideal forces, R, must respectively close.

The second condition which must be satisfied by the secondary stress lines is evidently derived from the rigidity of joints. This is accomplished by the use of the "deformation contour," a detailed description of which is given later.

V. THE EQUILIBRIUM CONTOUR

In a graphic analysis of secondary stresses the two conditions for locating secondary stress lines must be satisfied simultaneously at every step of the procedure. This necessitates the drawing of a great number of force diagrams required for each joint of the truss, on account of the continuously changing positions of the secondary stress lines. From a practical point of view, therefore, the conception of secondary stress lines as established in the last section is not suitable *during* the process of construction, for a speedy method demands that the direction of these lines be kept constant and that only one force diagram be required for each joint. This, however, does not mean the abandonment of the secondary stress lines, for at the end of the procedure these lines must be drawn in for every member of the truss, to show the distribution of secondary stresses.

The lines representing forces S and having a fixed direction for each member will be called hereafter the "section lines" to distinguish them from the secondary stress lines. The fixed direction of these section lines is best chosen perpendicular to the axis of the member, for then the offset (equal to f) would be found along the axis. For example, let S_{12} cut member 12 at c, Fig. 10. Stress f_{12} is then equal to 1 c, positive if c is between 1 and 2 and negative if on the other side of 1. For end 2, the same force S_{12} may be shifted to a point the distance of which from the end gives stress f_{21}. The direction of the section lines should be such that they give a counter clockwise moment for the joint whenever the value of f is positive; that is, when the lines cut the members somewhere between the joints. Thus, in the above example, the direction of the section line for end 1 should be upward and for end 2 downward. Note that these directions are constant for both positive and negative values of f, being always upward for end 1, whether on the right or left side of end 1—similarly for the line for end 2.

Consider a joint as shown in Fig. 11. As the section moduli of the members are given, a force diagram may be drawn of the section lines, with their directions perpendicular to the respective members.

Since a counter clockwise moment gives a positive f, the lines for the successive members should form a contour around joint 1 in a counter clockwise direction, as shown. The closing line of the force diagram, fe, corresponds to the external ideal force, R, for the secondary stress lines considered in the preceding section and must therefore be applied through joint 1, for the external moment about that joint is zero. Since the direction of R is constant, it may be drawn on the truss diagram as a part of the truss.

Since the condition of equilibrium of joints requires that the equilibrium polygon for section lines, S, and external ideal force, R, be closed for the joint, the stress in any one member may be found if it is known for all the other members meeting at the same joint. For example, suppose in Fig. 11 the stresses in all members are known except that in member 16. The section lines for members 12, 13, 14, and 15 may therefore be located from the values of f. The direction of the section line for member 16 is known from the force diagram and is seen to be upward. Its exact location on the truss may be found from an equilibrium polygon by taking some point as the pole and drawing the rays. To facilitate the construction, take f as the pole and draw the rays fb, fc, and fd. At the intersection, r, of forces FA and AB, draw a line parallel to fb, meeting force BC at s. Through s draw a line parallel to fc, meeting force CD at t. Through t draw a line parallel to fd, meeting force $R=FE$ at v. Then the section line for member 16 must pass through v. Drop a perpendicular from v on side 12, the prolongation of side 16. The value of f_{16} is then given by the distance $1w$. It is seen to be negative as it is on the opposite side of 1 from 6.

By a similar process the stress in any member at any joint may be found if the stresses in all the other members meeting at the same joint are known. The contour formed by the lines parallel to the rays in the force diagram will be known as the "equilibrium contour," as $prstv$ in Fig. 11.

If there are eccentric connections at the joint the moment thus produced may be taken care of by giving force R an offset equal to the moment divided by the value of R.

VI. THE DEFORMATION CONTOUR

1. Deformation and Property Lines

Consider equation (10) applied to vertex l of triangle plm, in Fig. 12.

$$r_{lp} = (U_{lm} / U_{lp})\, r_{lm} + K_{plm} / U_{lp}.$$

As this is a linear equation connecting r_{lp} and r_{lm} it may be represented by a straight line, ab, if sides, lp and lm, are considered as the co-ordinate axes of r_{lp} and r_{lm}. Positive values of r_{lm} will be laid off toward m from l with negative values in the opposite direction. Similarly for r_{lp}. To locate the line ab the simplest method would be to find the intercepts on the two axes.

$$\text{If } r_{lm} = 0,\ r_{lp} = K_{plm} / U_{lp}.$$
$$\text{If } r_{lp} = 0,\ r_{lm} = K_{plm} / U_{lm}.$$

That is, the intercepts on the two adjacent sides are respectively equal to the value of K_{plm} divided by the values of U for the corresponding sides. The signs of the intercepts are determined by the fact that if K is positive the positive value of K / U is found on the side which is first met with by passing around joint l in a counter clockwise direction; and the negative value of K / U on the side next met with in the same direction. Accordingly the value of K_{plm} / U_{lp} is laid off toward p while that of K_{plm} / U_{lm} is laid off away from m, as shown. This is for positive K; if it be negative the process should be reversed. From the intercepts thus obtained, the straight line, ab, may be located by passing through their ends. This gives the relations between r_{lp} and r_{lm} as represented by equation (10). For any value of r_{lm}, to find the corresponding value of r_{lp} it is necessary only to draw a line from the end of segment r_{lm} parallel to side lp, and from the intersection of this line with ab draw a line parallel to side lm. The intercept thus obtained on side lp gives the value of r_{lp} required.

By a similar method, a line cd may be drawn to represent the relations between r_{pl} and r_{pm}, positive values of r_{pl} being measured from p toward l and negative values away from l. A third line, ef, may

likewise be drawn for the vertex, m, of triangle plm. It is therefore evident that each triangle of a truss has three lines to represent equation (10) and they completely determine the deformation equation of Section II. These lines will hence be known as the "deformation lines" for the vertices of the triangle and will be designated in the figure by a letter denoting the triangle and a numeral denoting the joint. Thus, K_{A1} would designate the line which belongs to triangle A and joint 1.

It will be seen from equation (10) that if $K_{plm} = 0$, these deformation lines will pass through the origin of the axes, that is, the vertex of the triangle—joint 1 in this case. These lines will be known as the "property lines" of the truss since they represent the ratios of U which are the property of the truss. In other words, they are independent of the loading.

One feature of the property lines, which deserves attention, is that in any triangle the three property lines must meet in one point. This furnishes a good check on the deformation lines as they are parallel to the property lines.

2. The Deformation Contour

Suppose that, by the method just described, the deformation lines are constructed for all those triangles of a truss which have a common vertex; as, for example, those at joint 5, Fig. 13. Then, if the value of r_{53} be known, that of r_{52} may be obtained from line K_{B5} by drawing lines parallel to sides 53 and 52. Similarly, from the value of r_{52} thus obtained, that of r_{54} may be found from line K_{C5}. By continuing the same process r_{57} is eventually obtained with the aid of lines K_{D5} and K_{E5}. Hence—and this is important—if the value of the deformation modulus, r, of any member at any joint be known, it is known for all the other members at the same joint by the use of the deformation lines.

In order to check the result and also close the polygon, a deformation line, K_5, will be drawn for joint 5, giving the relationship between r_{53} and r_{57} considering the change in the angle that is outside of the truss, or the negative of the total changes in angles 352, 254,

456, and 657. Since both r_{53} and r_{57} are on the same straight line this deformation line K_5 could not be obtained by the usual Cartesian co-ordinates, but must be found as follows (see page 18): Find a pair of corresponding values of r_{53} and r_{57} and at the end of each segment draw a 45-degree line directed toward the other. These two lines will intersect in a point. This, with another point obtained in a similar manner, will then determine the deformation line K_5. For a joint where the outside members do not lie on the same straight line, the deformation lines may be obtained in the usual way. (See joint 2, Plate III).

The continuous broken lines (including the two 45-degree lines), drawn for each joint of the truss and parallel to the members, with their intersection points on the deformation lines, form a closed figure known as the "deformation contour." In Fig. 13 the deformation contour for joint 5 and r_{53} is *abcdefghjk*. For any other value of r_{53} the deformation contour for joint 5 will be parallel to the one as shown. That deformation contour which gives correct values of r, and consequently the actual secondary stresses, is known as the "correct deformation contour."

The deformation contour entirely expresses the deformation equation of Section II, and the fact that it must close for any joint forms the second condition imposed on the location of the secondary stress lines. In the graphic analysis, therefore, *the two fundamental equations are replaced by the closing requirements of the equilibrium and deformation contours.*

To fix the idea of the deformation contour it is necessary to give it a value equal to the deformation modulus, r, of one of the members meeting at the joint. For reasons stated later this member will be chosen as the chord at the left side of the joint in consideration. This will be known as the "reference member for the deformation contour." Thus the value of the contour for joint 5 just considered will be designated by r_{53}, 53 being the reference member for joint 5.

In order to utilize the equilibrium and deformation contours for the graphical analysis of secondary stresses, a relation will now be established which forms the basis of the method of deformation contour. Let a truss shown in Fig. 14 be analyzed for the secondary stresses.

Beginning with triangle A suppose the deformation contours for joints 1 and 2 are known, and let it be required to locate the deformation contour for joint 3.

From equation (14), $r_{31} = 3f_{13} - 2r_{13}$,

where r_{13} is known from the given contour.

Since $f_{13} = -(S_{12} / S_{13})\, f_{12}$, by equation (1)

$f_{13} = -(S_{12} / S_{13})\,(2r_{12} + r_{21}) / 3$. by equation (13)

Also, by equation. (10), $r_{13} = ar_{12} + b$,

where a and b are constants.

Therefore, $r_{31} = -(S_{12} / S_{13})\,(2r_{12} + r_{21}) - 2(a\, r_{12} + b)$.

That is, the value of r_{31} may be found from r_{12} and r_{21} by a linear equation, or,

$$r_{31} = m\, r_{12} + n\, r_{21} + p,$$

where m, n, and p are constants.

Next, consider the relations between r_{53} and r_{35}. By equation (14),

$$r_{53} = 3f_{35} - 2r_{35}.$$

Since

$$\begin{aligned} f_{35} &= -(S_{23} / S_{35})\, f_{32} - (S_{13} / S_{35})\, f_{31} \\ &= -(S_{23} / S_{35})\,(2r_{32} + r_{23}) / 3 - \\ &\quad (S_{13} / S_{35})\,(2r_{31} + r_{13}) / 3, \end{aligned}$$

it is seen that r_{53} is a linear function of r_{35} and r_{21} because the relations between r_{32} and r_{31}; r_{13} and r_{31}; r_{31} and r_{35}; and r_{23} and r_{21} are all linear. Therefore,

$$r_{53} = m' r_{35} + n' r_{21} + p',$$

where m', n', and p' are constants.

Similarly, consider the relations between r_{42} and r_{24}.

$$r_{42} = 3f_{24} - 2r_{24},$$

$$\begin{aligned} f_{24} &= -(S_{25} f_{25} + S_{23} f_{23} + S_{21} f_{21}) / S_{24} \\ &= -(S_{25} / S_{24})\,(2\, r_{25} + r_{52}) / 3 - \\ &\quad (S_{23} / S_{24})\,(2\, r_{23} + r_{32}) / 3 - \\ &\quad (S_{21} / S_{24})\,(2\, r_{21} + r_{12}) / 3. \end{aligned}$$

Now the following relations are linear: r_{25} and r_{24}; r_{52} and r_{53}; r_{23} and r_{24}; r_{32} and r_{35} (and hence r_{32} and r_{53}); r_{21} and r_{24}; r_{12} and r_{13} (and hence r_{12} and r_{31}, r_{12} and r_{35}, and finally r_{12} and r_{53}). Hence the relations between r_{42}, r_{24}, and r_{53} are also linear and it follows that

$$r_{42} = m'' r_{24} + n'' r_{53} + p''.$$

where m'', n'', and p'' are constants.

By a similar process it can be shown that for any triangle of the truss a linear relation can always be exprssed between the values of r at the three vertices. In general, for any triangle plm (Fig. 7) of a truss,

$$r_{ml} = g\, r_{lm} + h\, r_{pm} + k \qquad (15)$$

where g, h, and k are constants and r_{pm} is the deformation modulus for the reference member of joint p. By a successive application of equation (15) the value of the deformation contour for any joint of the truss may be found from two other given contours, say r_{12} and r_{21}.

By the aid of equation (15) the following principle may now be formulated:

"*In any framed structure composed of triangular elements the values of deformation contours for the joints of the structure are related to each other by a linear equation.*"* If the least number of members entering into a joint of the structure is n, the value of each of the deformation contours may be expressed linearly in terms of n others. In a bridge truss, n is generally equal to 2 so that *the deformation contours of any three joints of the truss are connected by a linear equation and, by the process of successive substitution, all the deformation contours of the truss can be made to depend, by linear expressions, on only two other deformation contours.*

The use of the above principle overcomes the necessity of solving a large set of simultaneous equations, which is always considered the most laborious part in the solution of secondary stresses. It is due to this principle that the methods presented in this paper take much less time than the four current solutions in general use.

*The effect of the primary stress on the deflection of the member is neglected.

Having thus completed the necessary discussions preparatory to the method of deformation contour, the graphic solution may now be applied to the analysis of secondary stresses.

VII. GRAPHIC ANALYSIS BY THE METHOD OF DEFORMATION CONTOUR

1. Graphic Representation of a Linear Equation involving Three Variables

In the previous section the values of deformation contours for any three joints in a trues are shown to have been connected by a linear equation. If r_{53}, r_{35}, and r_{21} are the contours at the vertices of a triangle 235, Fig. 15, then, by equation (15),

$$r_{53} = a\, r_{35} + b\, r_{21} + c.$$

If the value of one of the variables, say r_{21}, be assumed constant or arbitrarily assigned, the other two, r_{35} and r_{53}, can be graphically represented by a straight line. This, however, implies that the two variables are referred to two axes which are not in the same straight line as in the present case. Here, both r_{35} and r_{53} are referred to line 35. To overcome this difficulty and facilitate the construction the following device has been evolved.

Find a pair of values of r_{35} and r_{53} connected by the above equation, with r_{21} arbitrarily assigned, and locate the points a and a' by making $3a = r_{35}$ and $5a' = r_{53}$. From a draw a line at 45 degrees with 35 and directed toward the right of joint 3 and below the truss. Similarly from a' draw a line at 45 degrees with 53 and directed toward the left of joint 5, also below the truss. These two lines will intersect at a point A. Next find another pair of values of r_{35} and r_{53}, with the same r_{21} and locate the points b and b'. From b and b' draw lines at 45 degrees with 35 and directed toward each other, intersecting at B. Draw a straight line through A and B. This line will then give the relations between r_{35} and r_{53} for the assigned value of r_{21}. For any value of r_{35}, say $3c$, to find the

corresponding value of r_{53} draw a line from c inclined at 45 degrees with 35 and directed toward end 5. Let this line intercept the line AB at C. From C draw a line at 45 degrees with 35 and directed also toward 5. Let this intercept 35 at c'. Then r_{53} is given by the segment $5c'$. The above constructions can be easily proven by analytic geometry, as follows:

Let the equation between r_{35} and r_{53} with arbitrarily assigned value of r_{21} be

$$r_{53} = a\, r_{35} + f \qquad (a)$$

where $f = b\, r_{21} + c$ = constant. Let P, Fig. 16—the point obtained by the above construction with any two corresponding values of r_{35} and r_{53}—be referred to 35 and 3 5 as X and Y axes, so that its co-ordinates are x and y. From the figure,

$$y = \tfrac{1}{2}(d - r_{35} - r_{53}) \qquad (b)$$

where r_{35} and r_{53} carry their own signs. Also,

$$x = r_{35} + y. \qquad (c)$$

Substituting the value of r_{53} from (a) in (b) and eliminating r_{35} between the resulting equation and (c) there is obtained a relation between x and y:

$$y = \frac{a+1}{a-1}x - \frac{d}{a-1} + \frac{f}{a-1}$$

$$y = \frac{a+1}{a-1}x - \frac{d}{a-1} + \frac{b\, r_{21} + c}{a-1}$$

$$y = \frac{a+1}{a-1}x + \frac{b\, r_{21} - (d-c)}{a-1}. \qquad (d)$$

As this is a linear equation, the locus of P is a straight line.

It is evident from equation (d) that the slope of the line is a function of a alone, and independent of r_{21}. That is, the slope of the line is constant for all the assigned values of r_{21} and may be most readily obtained by making $r_{21} = 0$. This line will be known as the

"base line" for triangle 235, for the reason that it expresses the relation between the r's of base 35 with r at the vertex of the triangle = 0.

When the value of r_{21} is not zero the straight line given by equation (*d*) will occupy a new position parallel to itself with the y-intercept given by the following equation:

$$p = \frac{b}{a-1} r_{21} - \frac{d-c}{a-1} \qquad (e)$$

which is also linear and may be represented by a straight line. To construct this line proceed as follows: Prolong member 23 until it intersects the base line at *0*, Fig. 15. Assume a value for $r_{21} = 2e$, preferably as large as possible, and by the above method find line *EF* representing the relations of r_{35} and r_{53} for $r_{21} = 2e$. This line, as proven above, must be parallel to the base line, *AB*. From *e* draw a line parallel to member 23 and intersect *EF* at *E*. Join *OE*. This line *OE* then gives the y-intercept p for any value of r_{21}. If $2g$ is the value of r_{21} the intercept is found by drawing a line through g parallel to member 23 until it meets *OE* at *G*, and through *G* drawing a line parallel to the base line, *AB*. This line *GH* then gives the relation between r_{35} and r_{53} with $r_{21} = 2g$. The line *OE* giving relations between p and r_{21} at the vertex of the triangle, will be known as the "vertex line."

To utilize the base line and vertex line in finding r_{53} from r_{35} and r_{21} proceed as follows: Let $r_{21} = 2g$ (negative) and $r_{35} = 3h$. Through g draw a vertical line cutting the vertex line at *G*. Through *G* draw a line *GH* parallel to the base line, *AB*. Through h draw a downward, 45-degree line cutting *GH* at *H*. From *H* draw an upward, 45-degree line directed toward joint 5, cutting 35 at h'. Then r_{53} for the above values of r_{35} and r_{21} is given by $5h'$ (negative). Similarly, the values of r_{53} for any other combinations of r_{35} and r_{21} may be found.

By reversing the above process the value of r_{21} may be found from any given values of r_{35} and r_{53}. For these two values, say $3h$ and $5h'$, respectively, determine a point, *H*, by 45-degree lines and

through H draw a line parallel to the base line, cutting the vertex line at G. Through G draw a vertical line cutting member 21 at g. Then $2g$ is the value of r_{21} required.

By the same method the base line and vertex line may be constructed for any other triangle of the truss. In case the triangle has a base on the top chord of the truss, it is advisable to draw the 45-degree lines directed upward instead of downward as for member 35.

As a means of standardization all the base lines will be shown by heavy full lines and the vertex lines by heavy dashed lines.

2. Construction of Base Lines and Vertex Lines

Since the base line is a straight line it can be determined by two points. A point for the vertex line is given by the intersection of the base line with the vertical member produced, so that only one more point is necessary for the vertex line. Theoretically, therefore, three points are sufficient for constructing both the base line and the vertex line. For practical purposes, however, it is advisable to find one more point for each of the two lines—both as a check and as a means of improving the accuracy of the results. These points may be so chosen as to require the least amount of work.

Let the base and vertex lines be constructed for a truss as shown in Fig. 17. This is a Warren truss with verticals but the method is applicable to all kinds of trusses, the only difference being found in the order of procedure. This is governed by the number of members meeting at the joints. It is well to remark here that the base of a triangle is always chosen as the member which forms the outline of the truss.

(1) Base Line and Vertex Line for Triangle A

Here, either member 12 or 13 may be chosen as the base. While only one is necessary in the solution of the truss, it is advisable to have another in order to check the final results.

(a) Member 13 as Base

Since the vertex for triangle A is joint 2, the base line is obtained by taking $r_{21} = 0$, as 21 is the reference member for joint 2. To construct this base line three points are necessary and these are determined by the corresponding values of r_{13} and r_{31} connected by equation (14). The detailed process is as follows. Assume a value for r_{13} and complete the deformation contour for joint 1, giving r_{12}. Also, for $r_{21} = 0$, complete the deformation contour for joint 2, giving r_{21}, r_{23}, r_{25}, and r_{24}. From the values of r_{12} and r_{21} the stress f_{12} may be found by equation (13). Since equation (14) calls for $3f_{13}$ it must be found from f_{12}. This is best done by laying off $2r_{12} + r_{21}$ from joint 1—careful attention being given to the signs—and completing the equilibrium contour for joint 1, obtaining $3f_{13}$. From $3f_{13}$ subtract $2r_{13}$ and the difference will be r_{31}.

The value of r_{31} thus obtained, together with the assumed value of r_{13}, determines a point for the base line, use being made of the 45-degree lines as described in the previous article. By a similar process, still keeping $r_{21} = 0$, two more points may be obtained which, together with the one previously found, must lie on the same straight line. This gives the base line, *ab*, required.

To find the vertex line corresponding to the base line just found, keep one of the assumed values of r_{13} as a constant and find a pair of values of r_{21} and r_{31} which correspond. To do this, first give to r_{21} a value which is fairly large and complete the deformation contour for joint 2 with this value of r_{21}. The contour for r_{13} has already been drawn for constructing the base line so that r_{12} and r_{13} are known. From these contours obtain $2\ r_{12} + r_{21} = 3f_{12}$ and from $3f_{12}$ obtain $3f_{13}$. Subtracting $2r_{13}$ from $3f_{13}$, r_{31} is obtained. This r_{31} and the assumed r_{13} determine a point. Keeping the value of r_{21} constant, find another pair of values of r_{13} and r_{31}, determining another point. These two points must lie on a straight line, *cd*, that is parallel to the base line. Through the end of segment r_{21} thus assumed, draw a vertical line cutting *cd* at *c*. Let the base line, *ab*, cut member 23 produced at *a*. Join *ac*. This, then, is the vertex line required.

These base line and vertex line are indicated in the figure, as "Triangle *A*, Base 13."

(b) Member 12 as Base

Here, joint 3 is the vertex of the triangle and the base line is obtained by taking $r_{31} = 0$, as 31 is the reference member for joint 3. To find a pair of values of r_{12} and r_{21}, first assume a value for r_{12}. For this r_{12} complete the deformation contour for joint 1, obtaining r_{12}. Find $3f_{13} = 2r_{13} + r_{31} = 2r_{13}$. Construct an equilibrium contour for joint 1, and $3f_{12}$ is found from $3f_{13}$. From $3f_{12}$ and $2r_{12}$, the value of r_{21} is obtained. This r_{21} and the assumed r_{12} determine a point on the base line, use being made of the lines inclined at 45 degrees with member 12. Similarly, two more points may be obtained and the base line, *ef*, is completely known. Find the intersection of this line with a line drawn through joint 3 and perpendicular to member 12, at *e*.

To locate the vertex line, assign a fairly large value to r_{31} and, keeping it constant, obtain two pairs of corresponding values of r_{21} and r_{12}. These two points must lie on a line, *gh*, parallel to *ef*. Find the intersection, *g*, of this line, *gh*, with a perpendicular to member 12 through the end of segment r_{31}, assumed. The vertex line is then given by a line drawn through points *e* and *g*.

These base line and vertex line are indicated in the figure, as "Triangle *A*, Base 12."

To find the corresponding values of r_{12} and r_{21} for any assumed value of r_{31} drop a perpendicular to 12 from the end of segment r_{31} and find the intersection, *g*, of this perpendicular with the vertex line, *eg*. Draw a line through *g* parallel to the base line, *ef*. This line then gives the relations between r_{12} and r_{21} for the assumed r_{31}, the corresponding values being given by 45-degree lines.

(2) Base Line and Vertex Line for Triangle B

The base for triangle *B* is member 35, and the vertex, is joint 2, so that the base line is obtained by making $r_{21} = 0$, the deformation contour for which has already been drawn. As before, assume a value

for r_{35} and find the corresponding value for r_{53}. These two values will determine a point on the base line. On account of the greater number of members entering into joint 3 the processs is a little more complicated than for triangle A but the principle will be the same.

For the assumed value of r_{35} construct a deformation contour for joint 3, giveing r_{32} and r_{31}. Since the value of r_{53} depends on that of f_{35}, which, in turn, depends on f_{32} and f_{31}, or indirectly, on r_{32}, r_{23}, r_{31} and r_{13}, it is necessary to find the value of r_{13} for the corresponding values of r_{31} and r_{21}, as r_{23}, r_{32} and r_{21} are all known. To obtain this r_{13}, use is to be made of the base line and vertex line for triangle A, base 13, precisely as previously explained. For this reason it is advisable to choose $r_{21} = 0$ instead of $r_{23} = 0$ for this triangle, for then the base line for triangle A can be used. Now that the values of r for members 13 and 23 are known, the sums $2\,r_{32} + r_{23} = 3f_{32}$, and $2r_{31} + r_{13} = 3f_{31}$, may be obtained and the value of $3f_{35}$ obtained from an equilibrium contour for joint 3. Then, r_{53} is found from the difference of $3f_{35}$ and $2r_{35}$. These corresponding values of r_{35} and r_{53} determine a point on the base line for triangle B. Similarly, two more points may be obtained and the base line is completely known. For the vertex line, give r_{21} a value which is fairly large—preferably the same value used in constructing the vertex line for triangle A, base 13. This changed value of r_{21} affects r_{23} directly and r_{13} indirectly, for then the base line for triangle A, cannot be used in finding r_{13} from r_{31} and r_{21}. Instead, the vertex line for triangle A should be used for the assumed value of r_{21}. With this exception, the remainder of the method is the same as already described.

(3) Base Line and Vertex Line for Triangle C

The base for triangle C is member 24, and the vertex is joint 5. The base line is therefore obtainted by making $r_{53}=0$. Since the value of r_{42} depends on 3 f_{24} and 2 r_{24}, the latter being arbitrarily assigned, it is necessary to find f_{24}, or indirectly, f_{25}, f_{23}, and f_{21}. These values of f are found from r_{25} and r_{52}; r_{23} and r_{32}; and r_{21} and r_{12}. The deformation contour for joint 2 for the assumed r_{24} gives r_{25}, r_{23}, and r_{21}, while that for joint 5 with $r_{53}=0$ gives r_{52}.

The only r's unknown, therefore, are r_{32} and r_{12}. To find r_{32}, first obtain r_{35} from the base line and vertex line for triangle B, from r_{21} and $r_{53}=0$, then complete the deformation contour for joint 3 for this value of r_{35}. The contour will then give r_{32}. Lastly, the value of r_{12} is found from r_{13}, which, in turn, is found from the base line and vertex line for triangle A, where r_{21} and r_{31} are known. With these values of r known, $3\,f_{25}$, $3\,f_{23}$, and $3\,f_{21}$ may be obtained by addition, and $3\,f_{24}$ by an equilibrium contour for joint 2. The resultant, $3\,f_{24}$, then furnishes r_{42} for the assumed $2\,r_{24}$. These two values of r determine a point on the base line for triangle C. By a similar method two more points may be obtained, still keeping $r_{53}=0$, and the base line is then completely located. As a means of facilitating the construction it is well to have one value of r_{24} which will give $r_{21}=0$.

The vertex line for triangle C is obtained in much the same way as for triangles A and B.

By a similar process the base lines and vertex lines could be located for all the triangles of the truss. For a triangle like E, where five members enter into joint 5, the process is necessarily complicated but that is about the extreme case that is likely to occur in ordinary trusses.*

It will be found in practice that, with the exception of the triangles at the two ends of the truss, the base line and the vertex line are always very close together and in many cases the vertex lines could be omitted entirely. In this case the base line is to be used for all values of r at the vertex.

For a symmetrical truss with symmetrical loading the base line and the vertex line need be constructed for only one half of the truss, as for the other half they are identical in form. If the truss or the loading, or both, are not symmetrical the base line and the vertex line must be obtained for every triangle of the truss.

*The amount of work required is proportional only to the number of members entering the joint, and not to the size of the truss.

A few words are now necessary regarding the selection of the reference members. They are seen to be the chord members, end posts included, that are at the left side of the joints under consideration. This is due to the fact that the above process of constructing base lines and vertex lines advances from left to right and only a part of the deformation contour that is on the left side of the joint need be constructed.

3. Solution of the Problem

Let the base lines and vertex lines for all the triangles of the truss be constructed by the methods of the previous article. At the right end of the truss there are then drawn the base lines and vertex lines for triangles B', C', etc. For triangle A' three sets of base and vertex lines are drawn; first, for side $1'3'$ as base; second, for side $1'2'$ as base, considering equilibrium about joint $1'$; and, third, also $1'2'$ as base but considering equilibrium about joint $2'$. These three sets of lines are indicated as $1'3'$, $1'2'_{1'}$, and $1'2'_{2'}$ in the figure. Since they all represent the same relations among $r_{3'1'}$, $r_{1'2'}$, and $r_{2'1'}$ they may be used for solving the three unknowns as, analytically, they represent three equations. To perform this solution, first assume a value for $r_{3'1'}$ and apply it to sets $1'2'_{1'}$ and $1'2'_{2'}$, giving two lines parallel to their respective base lines. These two lines intersect at a point which will give a pair of values of $r_{1'2'}$ and $r_{2'1'}$ for the assumed $r_{3'1'}$. Next apply to set $1'3'$ the values of $r_{1'2'}$ and $r_{2'1'}$ thus found, and find the corresponding value of $r_{3'1'}$. This value should check with that previously assumed; if not, another trial may be made. Repeat the process until the values of $r_{3'1'}$, $r_{1'2'}$, and $r_{2'1'}$ all satisfy the three sets of base and vertex lines. These are the values that will give the correct secondary stresses. From the value of $r_{2'1'}$, that of $r_{2'4'}$ may be found from a deformation contour for joint $2'$. Similarly, the value of $r_{3'5'}$ may be found from $r_{3'1'}$ by a deformation contour for joint $3'$. These two contours are the correct deformation contours for the problem, since from them the correct secondary stresses may be obtained. From the values of $r_{2'4'}$ and $r_{3'5'}$, that of $r_{5'3'}$ may be obtained from the base line and vertex line for triangle B', and, consequently, the correct deformation contour for joint $5'$.

Repeating the process the correct contours are obtained for all the joints of the truss. To check the accuracy of the results the values of r_{31}, r_{12}, and r_{21} must satisfy the base line and vertex line for triangle A with member 12 as base, as this set of lines has not been used in constructing the correct deformation contours. This insures the correctness of every step in the procedure and is one of the important features of the method of deformation contour.

From the correct deformation contours the actual secondary stresses may be found by equation (13). This may be accomplished graphically by adding to segment r_{ln} of member ln a distance equal to r_{ln} plus r_{nl}, with proper signs, and measuring off the resulting segment with a scale three times as large as that used for r. It will be seen that on account of the large scale used for measuring f, the result could be read in three significant figures—a result accurate enough for all practical purposes.

When the loading and truss are symmetrical, the stresses at the two ends of the member of symmetry will be zero, hence the values of r at those two ends are also zero. This locates two correct deformation contours for the two joints of symmetry, from which the problem may be solved as before.

To obtain a clearer view of the distribution of secondary stresses in the truss, and also the way they affect each other, it is advisable to draw in the secondary stress lines for the different members and their force diagram, as mentioned in Section IV.

4. Checks

The following checks are found in this method of deformation contour:

(1) In constructing the values of K, the sum of the K's in each triangle must be zero.

(2) All the property lines in a triangle must meet in one point.

(3) All the deformation lines must be parallel to the property lines in the same triangle.

(4) All deformation contours must close for any value of r.

(5) All equilibrium contours must close.

(6) The base line and vertex line for any triangle are checked by constructing one extra point for each line. These extra points must lie on the respective lines.

(7) The correct deformation contours are checked by the base line and vertex line for the end triangle of the truss, with the end post as base.

(8) For any joint of the truss the equilibrium polygon for the secondary stress lines, S, and external ideal force, R, must close.

VIII. DETAILS OF GRAPHICAL CONSTRUCTION

The details of graphical construction for figures discussed in previous sections will now be given. To obtain a clearer view of the methods, constant references should be made to the drawings shown in Plate III.

1. Preliminary Considerations

(a) Scale

From equations (6) and (9) it is evident that both f and r are of the same dimensional degree as that of K/U, or $(6E\ \delta A)\ /\ (L/y)$, and the values of f and r should be measured off by a scale the same as the one used in constructing the deformation lines. Since L is always expressed in feet and y in inches, L and y may be represented by the same unit if $6E\ \delta A$ is changed into $1/2\ E\delta A$. Hence, with the understanding that y is expressed in inches but represented as if it were given in feet, the value of K should be understood as $E\ \delta A/\ 2$, instead of $6E\ \delta A$. This modified value of K will be used throughout the example worked out in next section. As the value of $E\ \delta A$ is found from the primary stresses by equation (2) the value of K will be given directly if only one half of the primary stress is considered.

The scale of S, the section modulus, used in constructing force diagrams may be anything independent of f, as it is only the direction that is required.

(b) Sign

In the present paper there are two instances where a conception of sign is necessary. One is rotational and the other is linear. The first applies to moments, angles, circular sections, and directions of contours while the second applies to the secondary stress lines, their offsets, f, deformation moduli, r, and the co-ordinates of variables. The rotational direction will always be considered positive if it is *counter clockwise*. For linear directions, with the exception of secondary stress lines and their offsets, a segment ax (Fig. 18) measured on the line ab from a to x, is always positive if x is laid off from a toward b, whether x is inside of ab or outside of b, in the latter case ax is greater than ab. Should ax be laid off from a away from b the segment will be negative. Similarly, if x is measured from b.

For the sake of uniformity all secondary stress lines will be considered as compressive stresses in the members. Under this convention if the lines in members 12 and 13 are as shown in Fig. 9, f_{12} and f_{21} will be negative (the moment is clockwise about 1 and also about 2) and f_{13} and f_{31} will be positive (the moments about 1 and 3 are counter clockwise). Hence, if the positions of secondary stress lines are known, both the sign and magnitude of f are at once determined. The value of f is for that fiber of the member where the stress has the same sign as the moment.

(c) Truss Diagrams

Two truss diagrams are necessary in this method. One of them is used to record the given information and to construct the values of K while the other is for the deformation lines and the solution of the truss. On each of these diagrams there should be drawn certain figures that are preparatory to the analysis: namely, the "y-circles" and "U-lines" for the first diagram and the property lines for the second diagram.

The y-circle is drawn at the end of a member, with a radius equal to the value of y of the member and measured off with a scale equal to that used in constructing the truss diagram in feet, although the value

of y is expressed in inches. From the other end of the member draw a line tangent to this circle. This line will give the ratio of $L/y = U$ and will be known as the U-line. To find the ratio of K/U if K is known, lay off a segment equal to K from the end of the member from which the U-line is drawn. At the end of segment K draw a circle tangent to the U-line. The radius of this circle then gives the ratio required.

To construct the property lines in the second truss diagram, use is made of the U-lines in the first diagram. Suppose the property line in triangle A and joint 1 is required (Plate III). On the first diagram where the U-lines are drawn describe any arc with joint 1 as center, cutting member 12 at a and member 13 at b. It is preferable to make the radius of the arc as large as possible in order to secure accuracy. With a and b as centers draw two arcs tangent to the respective U-lines. Let the radius having a as center be called u_{12} and that having b as center be called u_{13}. On the second truss diagram, in triangle A, lay off the segment u_{12} on side 13 (not side 12), letting the end point be c. With c as center draw an arc with radius equal to u_{13}. Through c draw a line parallel to side 13 cutting the arc at d. Join d to joint 1. This, then, is the required property line for triangle A, joint 1. Similary, all the other property lines in triangle A may be obtained. As a check, these three property lines should meet in one point. This point is enclosed in a small circle in Plate III.

Exactly as described before, the y-circles and U-lines may be constructed for every member of the first diagram, and the property lines for every triangle of the second diagram.

2. Force Diagram for Equilibrium Contour

As mentioned before, the force diagram for equilibrium contours need be drawn only once for the whole process as the directions of the section lines are constant, being perpendicular to the respective members. For this reason it is advisable to construct the force diagram before the solution of the truss, in order to record the values of S in a graphical form. In solving a truss as shown in Plate III first

draw a line normal to member 12 and measure off $ga=S_{12}$. Then from a draw a line normal to member 23 and measure off $ab=S_{23}$. From b draw a line normal to member 25 and measure off $bc=S_{25}$. Continue for all the web members of the truss until points e and f are obtained. The directions of the forces are determined by the fact that the section lines around any joint should form a contour in a counter clockwise direction, if all the values of f are positive. Only one half of the truss is considered here as the other half will be symmetrical. Now from a draw a line normal to member 13 and make $ah=S_{13}$. The point h should be on the under side of a so that force ah will produce a counter clockwise moment about joint 1 for positive f. Similarly, from points b, c, d, and e draw lines perpendicular to the respective chord members and measure off the segments bk, cj, etc., equal to S_{35}, S_{24}, etc. Next join the end points l, j, g, h, etc., with dotted lines as shown. These will then give the directions of the external ideal forces, R, which are to be applied through the corresponding joints in the truss. Lastly, the rays gb, gc, etc., are completed. They are the lines which join the left end of the external force, R, to the inner points of the diagram.

3. Construction for values of K

From equation (2),

$$E\,\delta A_{213}=P_{23}\,(\cot 132+\cot 123)-P_{13}\cot 132-P_{12}\cot 123,$$

$$E\,\delta A_{132}=P_{12}\,(\cot 123+\cot 213)-P_{23}\cot 123-P_{13}\cot 213,$$

$$E\,\delta A_{321}=P_{13}\,(\cot 213+\cot 132)-P_{12}\cot 213-P_{23}\cot 132.$$

In triangle 123, Fig. 19, through 1 draw a line perpendicular to side 23 with a length equal to P_{23} and through its end draw a line parallel to 23, giving segments a and b. Similarly find the segments c, d, e, and f. Then,

$$E\,\delta A_{213} = (a+b)-(f+d),$$
$$E\,\delta A_{132} = (c+d)-(a+e),$$
$$E\,\delta A_{321} = (e+f)-(c+b),$$

or, the change of angle 213 is equal to the sum of segments a and b belonging to this angle, minus the sum of the segments of the other two

angles, f and d, which are not adjacent to those of the angle under consideration—that is, a and b.

To secure a systematic arrangement both in sign and magnitude the following scheme is advisable. The example is for a triangle with a right angle but the method could easily be extended to any other kind of triangle. Lay off the primary stresses per square inch as shown in Fig. 20. Parallel to the sides of the triangle draw lines through the ends of the segments, obtaining a, b, c, and d, which are of the same signs as the corresponding primary stresses. There are thus obtained three small triangles 13_12_1, 21_23_2, 32_31_3, similar to the large triangle 123. The consecutive sides of the small triangles may now be followed as a contour in a certain adopted direction, say counter clockwise, as shown. The change of angles may then be found as follows, taking as an example, 213, Fig. 20: At end 2_1 of segment a, met with in a counter clockwise direction after 3_1 when following the contour 13_12_1, measure off 2_1A1 equal to the segment c of triangle 32_31_3 which is adjacent to side 23—the side parallel to 2_13_1. If the sign of c is the same as that of a it is laid off from 2_1 toward 3_1, but if otherwise, it is laid off in the direction away from 3_1, as shown in the figure. Then the distance 3_1A1, the segment from the initial point 3_1 to the end point $A1$, gives the value of $E\delta A_{213}$. The sign of 3_1A1 is the same as that of a because $A1$ is reached from 3_1 in a counter clockwise direction. In case c and a have the same sign and c is greater than a, then segment c is laid off from 2_1 toward 3_1 and point $A1$ is found below 3_1. The value of $E\delta A_{213}$ is then opposite in sign to that of a, because the point $A1$ is reached from 3_1 in a negative direction, clockwise, when following tho contour 13_12_1. Similarly the change in angle 123 may be found from the segments b and d, the latter being that part of side 1_32_3 of triangle 31_32_3 which is adjacent to the side of the large triangle parallel to 1_23_2.

To find $E\delta A_{132}$ lay off segments a and b from end 1_3 of side 1_32_3, care being taken of the signs. If a or b, or both, are of the same signs as c and d the said segments should be laid off toward 2_3, instead of away from 2_3, as shown, which is for a and b having different signs from c and d. The rest of the procedure is the same as for the other two angles.

Since the sum of the changes of angles in a triangle must be zero, the values of $E\,\delta A$ thus found are subject to a unique check.

As mentioned before the value of K is taken as $E\,\delta A\,/\,2$ on account of the y-circles being magnified 12 times. To give this directly from the change of angle, the primary stresses may be laid off on a scale that is one half as great as that used in measuring the segments 3_1A1, etc, for then $3_1A1 = K_{A1}$; $1_2A2 = K_{A2}$, and $2_3A3 = K_{A3}$.

4. Deformation Lines

From the values of K thus obtained, the ratios of K/U could be found from the U-lines as explained in Article 1, (*c*). On the second truss diagram where the property lines have already been drawn, lay off the values of K/U on the corresponding sides, with proper regard to the signs. The deformation lines may then be drawn through the ends of these segments. They should be parallel to the property lines drawn in the same triangle. In Plate III the deformation lines are indicated by a shaded triangle which the deformation lines formed for the triangle of the truss.

5. Base Lines and Vertex Lines

The detailed descriptions for base lines and vertex lines have been given in Section VII, Article 2. As a preliminary it is necessary to construct the deformation contour, the graphical addition of $3f_{lm} = 2r_{lm} + r_{ml}$, the equilibrium contour and the graphical subtraction of $r_{ml} = 3f_{lm} - 2r_{lm}$. The two contours have been considered in detail in previous articles so that the only explanations necessary are for the graphical addition and subtraction. To perform $2r_{lm} + r_{ml}$, first find the value of $2r_{lm}$ by adding to r_{lm} a segment equal to itself, and then adding or subtracting r_{ml}, depending on its sign. Attention is here called to the discussion on linear directions in Article 1, (*b*). To obtain $3f_{lm} - 2r_{lm}$ first find $2r_{lm}$ as just described. Since both r_{lm} and f_{lm} are referred to joint l as origin, the difference is then given by the distance *from* the end of segment $2r_{lm}$ *to* the end of segment $3f_{lm}$ (this is given by the intersection of section line lm and member lm), both in sign and in magnitude. It is positive if the segment is in the direction from l to m, and negative if from m to l.

6. Secondary Stress Lines

From the values of correct deformation moduli secondary stresses, f, may be obtained from equation (13) by graphical addition. More directly, the segments $2r_{lm}$ and r_{ml} may be added to each other and the resulting segment measured with a scale three times as large as for r, or for K/U. From the values of f thus obtained, the offsets of secondary stress lines are therefore known and the latter may be drawn in by tangent to arcs drawn with the values of f as radii. From the directions of secondary stress lines, a force diagram of S may be drawn to show the effect of section moduli upon the secondary stresses.

IX. EXAMPLE

To illustrate the method of deformation contour a truss will now be analyzed for secondary stresses with the given values of P and S as shown in Plate III. This is the same problem as that solved by Kunz in illustrating the method of Mohr in Engineering News, vol. 66, p. 397, and by Turneaure in illustrating the method of Manderla in Engineering News, vol. 68, p. 438. A fair comparison can therefore be made of the three methods, and their relative merits determined therefrom.

X. CONSTRUCTION OF INFLUENCE LINES

In constructing influence lines the only change that is necessary in the above method is found in the value of K. Since in equation (10) the coefficient of r_{lm} is a constant for the truss, the slope of the deformation lines which represent the equation is a constant and the deformation lines for all kinds of loading in the same triangle and for the same joint are parallel to each other. Similarly in equation (15) the values of g and h are also constants for the truss and the base line and vertex line for any triangle are respectively parallel to each other for different kinds of loading. If the said lines are drawn for any one kind of loading, the slopes of these lines will be known and for each additional loading only one extra point is necessary for determining the lines. The present method is therefore very well suited to the construction of influence lines. If Müller-Breslau's method of keeping a constant left hand reaction be used, the method can be made still simpler.

ANALYTIC SOLUTION OF SECONDARY STRESSES WITH SCIENTIFIC ARRANGEMENT OF COMPUTATIONS

XI. GENERAL PROCEDURES

From the equations derived in Section III and the principle of deformation contour discussed in Section VI an analytic solution may be made of the secondary stresses without the use of a large set of simultaneous equations, as required in the ordinary methods.

First of all let the graphical construction of deformation contours be reduced to an analytic form. Apply equation (10) to joint 5 of Fig. 13;

$$r_{52} = (U_{35} / U_{25})\, r_{53} + K_{B5} / U_{25},$$
$$r_{54} = (U_{25} / U_{45})\, r_{52} + K_{C5} / U_{45},$$
$$r_{56} = (U_{45} / U_{56})\, r_{54} + K_{D5} / U_{56},$$
$$r_{57} = (U_{56} / U_{57})\, r_{56} + K_{E5} / U_{57}.$$

By successive substitution,

$$r_{57} = (U_{35} / U_{57})\, r_{53} + (K_{B5} + K_{C5} + K_{D5} + K_{E5}) / U_{57},$$

or, in general,

$$r_{lp} = (U_{lm} / U_{lp})\, r_{lm} + \Sigma K / U_{lp}, \qquad (16)$$

where ΣK is the sum of K's for angles successively met with in passing around joint l in a clockwise direction, from member lm to member lp. It should be negative if member lp is met with after member lm in a counter clockwise direction. Equation (16) takes the place of the deformation contour in the analytic solution and is to be used in finding the deformation modulus of a member at any joint when it is known for another member at the same joint. For convenience, this process is oftentimes spoken of as passing a deformation contour around the joint.

The general procedure for the analytic method is best understood by applying it to a truss as shown in Fig. 14. Since the principle of deformation contour states that the deformation modulus for any member in a bridge truss can be expressed in terms of two others, let

these moduli be for member 12 and taken as two unknowns; or, more directly, take $f_{12} = x$, and $f_{21} = y$. Then by equation (9), $r_{12} = 2x - y$, and $r_{21} = 2y - x$. From these two values of r find those for the other members meeting at joints 1 and 2, by equatien (16), as the values of U and K are easily computed from the given information. Applying equilibrium equation (1) to joint 1, f_{13} may be obtained from x, giving the stress at one end of member 13. To find the stress at the other end, f_{31}, first subtract f_{13} from r_{13}, obtaining $f_{13} - f_{31}$ ($r_{13} = 2f_{13} - f_{31}$), and then subtract $f_{13} - f_{31}$ from f_{13} giving f_{31}. Subtracting again $f_{13} - f_{31}$ from f_{31}, $r_{31} = 2f_{31} - f_{13}$ is obtained. From r_{31} thus found obtain r_{32} and r_{35} by equation (16). Now, for member 23, the values r_{23} and r_{32} are known and therefore the stresses f_{23} and f_{32} can be found from equation (13). At joint 3 the stresses in all members are known except that in 35, which may be found by an equilibrium equation around joint 3:

$$S_{31} f_{31} + S_{32} f_{32} + S_{35} f_{35} = 0.$$

By a process similar to that for member 13, there is obtained r_{53}, from which, by equation (16), are determined the values of r for all the members meeting at joint 5.

The stresses in member 25 may now be found from r_{25} and r_{52}. This makes known all the stresses around joint 2 except that of member 24 which can be found from the equilibrium equation. From the value of r_{24}, that of r_{42} may be found from f_{24} by the same method as for member 13. A deformation contour can therefore be passed for joint 4 with this value of r_{42}. The above process may be repeated for all the members of the truss until, eventually, the stresses in members around joints 2′ and 1′ are obtained from the equilibrium equations for joints 2′ and 1′. Up to this point the quantities $K_{A'1'}$ and $K_{A'2'}$ have not been used and it is necessary to test the deformation equations involving these two quantities. Applying equation (5) to joints 2′ and 1′ of triangle A', there are obtained two equations which furnish the means for solving x and y. After they are obtained, all the other stresses being expressed in terms of x and y may be found by substitution.*

*See also Section XIV, p. 44.

If the loading is symmetrical about a member at the center of the truss, the stresses at the two ends of that member will be zero. Since these stresses are expressed by linear equations in terms of x and y, the latter can be found by making the stresses equal to zero.

In the analytic method of solution the following points should be noted:

(1) In applying equation (16) to members which are reached from the reference member lm in a counter clockwise direction within the truss the sign of ΣK should be reversed.

(2) The equilibrium equation cannot be applied to a joint until the stresses at the joint are known in all members except one.

(3) Apply equation (5) to any triangle, plm, (Fig. 7);

$$U_{mp}(2f_{mp} - f_{pm}) = U_{lm}(2f_{ml} - f_{lm}) + K_{lmp},$$
$$U_{lp}(2f_{pl} - f_{lp}) = U_{mp}(2f_{pm} - f_{mp}) + K_{mpl},$$
$$U_{lm}(2f_{lm} - f_{ml}) = U_{lp}(2f_{lp} - f_{pl}) + K_{plm}.$$

Adding together, noting that ΣK should be zero in a triangle,

$$U_{lm}(f_{lm} - f_{ml}) + U_{mp}(f_{mp} - f_{pm}) + U_{pl}(f_{pl} - f_{lp}) = 0. \qquad (17)$$

This equation must be satisfied as soon as the stresses in a triangle are completely known. This is one of the important features of the analytic method.

(4) Particular attention should be paid in the solution of equilibrium equations and also in the correct substitution of the values of U. Mistakes made here could not be detected by equation (17).

(5) The values of f found from equation (13), $f_{lm} = (2r_{lm} + r_{ml})/3$, can be obtained in a simpler way as follows: First find $r_{lm}/3$ from r_{lp} of reference member lp, using $3U_{lm}$ instead of U_{lm}. This gives

$$r_{lm}/3 = (U_{lp}/3U_{lm})\, r_{lp} + \Sigma K/3U_{lm},$$

from equation (16). Similarly, find $r_{ml}/3$. Then obtain the sum of $r_{lm}/3$ and $r_{ml}/3$, which when added to $r_{lm}/3$ gives f_{lm}, and when added to $r_{ml}/3$ gives f_{ml}. Since only the stresses in web members are found from the values of r alone, only the web members need be computed for $3U$.

XII. SCIENTIFIC ARRANGEMENT OF COMPUTATIONS

As is well known, an analytic method while more accurate takes longer time. It is therefore necessary to devise some method by which the amount of work will be reduced to a minimum. The computations for secondary stresses by the ordinary methods have been systematized to a great extent in recent years, but there is still room for improvement, mainly in the arrangement of computations. Among the many items which are inefficient and are found in all of the ordinary methods may be mentioned the following:

(1) All these analytic methods need some graphic construction, one way or the other, to show the proper location of certain computed quantities.

(2) In all ordinary methods the computations are arranged in tabular form in which at least one or two columns of figures are copied from previous tables. This not only wastes time and energy but also invites error.

(3) Only the final results are shown in the tables, the rough computations and figuring, like addition and subtraction, are made on a separate paper, which is not filed, and are likely to be performed in an unsystematic way.

(4) Time is wasted in copying the results of rough figuring, thus increasing the liability of mistakes.

(5) There is no method of checking the computations except by repeating the figuring, which not only takes time but also requires a different computer.

(6) The figures arranged in a table do not convey as much idea as when marked on a truss diagram, as a semi-graphical method is always superior to analytic tabular forms.

To get rid of the above objections, which are very common, a simple and self evident scheme would be to dispense entirely with the idea of tables, and to record all the results on the truss diagram direct. A second thought makes it plain that to save time it is also advisable to include all the rough computations on the diagram. By this arrangement it is possible so to record the figures that none is written more

than once and the computations may be most readily performed. Further, by arranging the computations so that each figure has a definite space on the diagram, the chance of using incorrect figures is entirely eliminated.

As a first application of the scientific arrangement of computations let the changes of angles in a triangle as given by equation (2) be obtained on a truss diagram as shown in Fig. 21. First write down the values of P on the sides of the triangle and then insert these values within the triangle, each value being entered twice. Next form the differences, $P_{12}-P_{23}=a$, and $P_{12}-P_{13}=b$, and multiply each by the cotangent of the corresponding angle, which is written above the columns for stresses P. The sum of these products, c, and d, gives $E\,\delta C$. Next find $P_{23}-P_{13}=e$; also e cot $C=f$. Add f to negative of d, $E\,\delta B$ is obtained. Similarly for $E\,\delta A$. If C is a right angle f and h will be zero, then $E\,\delta B=-d$, and $E\,\delta A=-c$. This is the case shown in Plate IV.

XIII. EXAMPLE. DETAILS OF PROCEDURE

Let the truss analyzed in Section IX be used to illustrate this analytic method and also the scientific arrangement of computations. Refer to Plate IV.

On the upper portion of the drawing construct a small truss diagram to record the given data, and solve for the values of K. Construct another truss diagram with a scale as large as the drawing will permit. This diagram will be used for the analytic solution of the stresses. On each of the web members construct two small tables one with seven rows and the other three rows, each having three vertical columns and placed one above the other. In the first table extend the third and fifth rows one space to the right, as shown. For each of the chord members construct a small table of three columns and six rows with the second row extended one space to the right. For each of the joints construct two tables with one adjacent to the truss. The size of the table adjacent to the truss is determined as follows:

Number of vertical columns=number of members meeting at the joint.

Number of horizontal rows = number of vertical columns.

The other table for the joint has four vertical columns and as many horizontal rows as may be needed in the process.

The table drawn on the chord member and the large table on the web member will be known as *F*-tables. The two tables for the joint will be known as *K*- and *M*-tables, the former applying to the one adjacent to the truss. The small tables below the *F*-tables for the web members will be known as *C*-tables. Mark all the vertical columns by letters as shown in the first horizontal rows. For the *F*-tables these will be x, y, and k and for the *M*-tables, S, x, y, and k. For *K*-tables the identification marks for all the members entering the joint should be noted, except those for the reference members.

Next fill the *K*-table with the values of K found in the small truss diagram. For example, at joint 5, the value of K_{B5} is put in the second row and second column; that of K_{C5} in the third row and second column; that of K_{D5} in the forth row and third column; and, finally, that of K_{E5} in the fifth row and fourth column. There will then be one column vacant at the right of K_{E5}. At joint 2, the value of K_{A2} with its sign reversed, is put in the second row and second column; that of K_{B2}, sign reversed, in the third row and second column, and finally that of K_{C2}, with sign reversed, in the fourth row and third column. One column is left vacant at the right of K_{C2}. Similarly, all the *K*-tables may be filled with the values of K transferred from the truss diagram at the top of the drawing. Next there are formed the summations of the various values of K, as follows: Find the algebraic sum of the figures in the second column and write the result in the third column and third row. This figure is next added algebraically to the figure below, the result being put in the fourth column and fourth row. Similarly, this figure may again be added to the figure below if there is any, and, finally, the space at the lower right hand corner of the table is filled. Thus, at joint 5, the sum of -0.55 and -2.10 is -2.65, that of -2.65 and -0.75 is -3.40 and that of -3.40 and 0.94 is -2.56. Similarly, for the joints at the top chord. It will be seen that the first figure in the second column is the value of ΣK for the first member met with in

clockwise direction around the joint from the reference member, if the joint is at the bottom chord, and in a counter clockwise direction if the joint is at the top chord. Similarly, the first figure in the third column is the value of ΣK for the second member met with in the proper direction from the reference member, the first figure in the fourth column is that of the third member from the reference member, and so on. The proper members to which these sums apply are written down in the first row. Thus, at joint 5, the figure -0.55 is for member 52, -2.65 is for member 54, and so on.

Above and below the tables constructed on the members marked as F and C there are recorded the values of $U=L/y$, where L is in feet and y in inches, and $S=I/y$. For web members the values of 3 U are also recorded beside U.

If the various sums in the K-tables are divided by U, the constant term in equation (16) will be obtained for the respective members. As mentioned before, the value of $r/3$ is to be used for the web members and ΣK should then be divided by 3 U for the web members. These values of $\Sigma K/U$ and $\Sigma K/3U$ are recorded in the F-tables, in the extended portions provided at the right. Thus, for member 25 at joint 5, the value of $\Sigma K/3U$ from reference member 53 is $-0.55/15.43 = -0.036$ and is put in the extended portion of the fifth row in the F-table for member 25. This is the increment for r_{52}. Similarly, for r_{25} the value of $\Sigma K/3U$ from reference member 21 is $-2.04/15.43 = -0.133$ and is put in the extended portion at the right of the third row of the F-table for member 25. As another example, the value of $\Sigma K/U$ for chord member 57 at joint 5 from reference member 53 is $-2.56/2.93 = -0.84$. By the same process all the extended portions of F-tables will be filled up with the values of $\Sigma K/U$.

Now, to solve the problem, let $f_{12}=x$ and $f_{21}=y$, and put the proper figures in the F-table for member 12. Subtracting the coefficients of x and y for f_{21} from those for f_{12}, the difference $f_{12}-f_{21}$ is obtained. Reverse the signs of the difference and put them above the original signs. By using these reversed signs the difference $f_{21}-f_{12}$ is obtained. When f_{12} is added to $f_{12}-f_{21}$, the result is r_{12}, and if f_{21} is added to $f_{21}-f_{12}$, the result is r_{21}. In other words,

r_{12} is obtained by adding f_{12} to $f_{12}-f_{21}$ with the lower signs, and r_{21} is obtained by adding f_{21} to $f_{12}-f_{21}$ with the upper signs. With the value of r_{21} thus found, apply it to members at joint 2 by equation (16), as follows: Multiply the coefficient of x, -1, for r_{21}, by $U=4.38$ giving -4.38. Divide this by $3U$ of member $23=13.55$ and put the quotient, -0.324, in the column for x and row for $1/3\ r_{23}$ in the F-table for member 23. Divide -4.38 by $3U$ of member $25=15.43$ and put the quotient, -0.284, in the proper space in the F-table for member 25. Similarly divide -4.38 by U of member $24=3.03$ and put the quotient, -1.46, in the proper space for member 24. Since the rows for r have an extended portion at the right, there is no chance of recording the figures in the wrong places. By a similar process fill the columns for y in the F-tables for the members around joint 2. For values of constant k the same process is to be pursued except that the figures are put directly above the extended portions of the respective horizontal rows. Thus, k for $25=k$ for 21 multiplied by $4.38/15.43=0$, since k for member 12 is zero. This figure, 0, is then put above the extended portion of the third row. Now add the figures in the extended portion and the one above and write the algebraic sum in the same row in column k. Thus, $0-1.33=-1.33$ for member 25. Similarly, the constant term k is obtained for all the members around the joint and the deformation contour for joint 2 is then completely determined by analytical expressions. By the same method, complete the contour for joint 1.

To find f_{13} from f_{12}, apply the equilibrium equation to joint 1. Multiply by S_{12} the coefficients of x, y, and k of f_{12} and enter the products (equal to the moment due to f_{12}) in the M-table at joint 1. This part is believed to be self evident. Reverse the sign of the moment and enter the result in the row for member 13, whose S is equal to 155. Dividing the moment of 13 by S_{13}, f_{13} is obtained. Enter the result in the third row of the F-table for member 13. There are already in this table figures in the second row representing r_{13}. Subtract f_{13} from r_{13} giving $2f_{13}-f_{31}-f_{13}=f_{13}-f_{31}$. Thus, for coefficient of x, $2.995-(-2.403)=5.400$. Next, reverse the signs of $f_{13}-f_{31}$ and put the new signs below the old. Thus, for coefficient

of ϖ, ± 5.400. Next, subtract $f_{13}-f_{31}$ from f_{13} or, using the lower signs, add $f_{13}-f_{31}$ to f_{13}. Thus, the coefficient of ϖ is $-2.405-5.400=-7.805$. This gives the value of f_{31}. Again subtract $f_{13}-f_{31}$ from f_{31} giving $2f_{31}-f_{13}=r_{31}$, or, using the lower signs of $f_{13}-f_{31}$, add to it f_{31}. Thus the coefficient of ϖ for r_{31} is $-5.400-7.805=-13.205$. From the value of r_{31} thus obtained, a deformation contour around joint 3 may be obtained by analytic expressions as explained for joint 2, obtaining thereby $1/3\ r_{32}$ and r_{35}. Since for member 23 there are already computed the values of $1/3\ r_{23}$ and $1/3\ r_{32}$ in the *F*-table, the difference of these two quantities may next be obtained. This, as may be shown by subtracting equation (13), is equal to $f_{23}-f_{32}$. These figures are to be placed in the sixth row. Next find the sum of $1/3\ r_{23}$ and $1/3\ r_{32}$ and put the result in the fourth row. This quantity when added to that in the third row gives f_{23} in the second row, and when added to that in the fifth row gives f_{32} in the seventh row. All the stresses being completely known in the first triangle, it is necessary to apply the test of equation (17). Noting the direction of the contour by which the differences of f are taken, it will be seen that the figures in the sixth row of the *F*-table for 23, plus the figures in the fourth row of the *F*-table for 13, using the lower signs, plus the figures in the fourth row of the *F*-table for 12, using the lower signs, each multiplied by the corresponding value of U, should be zero. These figures are recorded in the *C*-table under the *F*-table for member 23.

Now, around joint 3 the stresses are known in all members except that in member 35, which can then be found from the equilibrium equation. This summation is performed in the *M*-table for joint 3. From f_{35} thus obtained and r_{35} already found, the *F*-table for member 35 may be filled as for member 13. From r_{53} obtained, complete the deformation contour around joint 5.

In the *F*-table for member 25, since $1/3\ r_{25}$ and $1/3\ r_{52}$ are known, it can then be filled as for member 23. As before, the figures in the sixth row of the *F*-table for 25, and those in the fourth row of the *F*-table for 35, using the lower signs, each multiplied by the proper U are next put in the first two rows of the *C*-table. The

last row in the table is copied from the first row of the C-table for member 23, with the signs reversed. The sum of the figures in each column of the table must be zero.

By the same process the stresses in all members are obtained in terms of x and y in linear equations. The two simultaneous equations derived from the fact that f_{67} and f_{76} should be zero furnish the means to solve x and y. From these values of x and y the stress f in any member may be found in the following manner: On the side of the member where the fiber stress is obtained; that is, on the side first met with in passing around the end in a clockwise direction from the outside, the values of the terms containing x and y are written one above the other after substitution. The constant k is next written on the side of these two figures toward the end of the member. Find the algebraic sum of the terms of x and y and write the result above k. Add this to k, algebraically, the result is the value of f, which is to be written near the end of the member.

XIV. CONSTRUCTION OF INFLUENCE LINES

Analogous to the relations between deformation contours there is also a linear law existing between secondary stresses f. Consider a truss composed of triangular elements. In each triangle there are then six values of f, one at each end of the member. But the triangles are connected together and for every pair of adjacent triangles there is one side in common. One of the triangles may be considered as added to the other by the introduction of two sides, and therefore has only four values of f. If the structure is built up of triangles by successive addition to a "base triangle" all of the added triangles will then have only four unknowns while the base triangle will have six. To find these unknowns there is required an equal number of equations which must be found from the nature of the problem. Let the number of triangles in the structure be n. If one of the triangles be considered as the base triangle and the other $(n-1)$ triangles be added to this triangle by successive introduction of two sides at a time, there will be $6+4\ (n-1)=4\ n+2$ unknowns. For each triangle there are three

angles giving three deformation equations, (3). This furnishes 3 n equations and only $n+2$ more are required. But this is equal to the number of joints in the n triangles and for each joint there is also an equilibrium equation, (1). Since both the deformation and equilibrium equations are linear in form it follows that all the secondary stresses are connected by equations linear in form.

Since the base triangle has six values of f and there are only four equations available, three from the deformation equations and one from the equilibrium equation for the joint that is free from other triangles, two of the values of f could not be found until two more equations are introduced. This base triangle therefore has two unknowns at the outset of the solution. As the stresses in the other triangles depend on those in the base triangle, all of them will be expressed in terms of only two unknown quantities. Hence if f be the stress in any member and f_x and f_y be the two unknowns in the base triangle then,

$$f = a\ f_x + b\ f_y + c,$$

where a, b, and c are constants. To determine f_x and f_y it is only necessary to notice that the very last triangle added to the structure has three joints but only one is used for the equilibrium equation. There are therefore two more equations expressing the moments at these two joints, which furnish the means to solve f_x and f_y.

An examination of the methods by which the above equation is derived (see Section XI) will show that the constants a and b are functions of U only while c is a function of both U and K. Hence a and b depend only on the truss dimensions and the sections of members while c depends in addition on the loading, as K is a function of the change of angles which depends on the loading. This fact shows that in constructing influence lines the only change in the analytic method that is necessary is to extend the various tables on the truss diagram so that there will be room for each change of loading which means a change of k.

XV. COMPARISON OF RESULTS

The secondary stresses worked out in the examples of this paper are compared with those obtained by the ordinary methods in a table shown on next page. The figures in the fifth and sixth columns of this table are taken respectively from Plates III and IV. The figures in the third column are obtained from the article in Engineering News, vol. 66, p. 398 and those in the fourth column are obtained from the article in Engineering News, vol. 68, p. 439. These two articles have been mentioned before as to contain the analyses of secondary stresses by the method of Mohr and the method of Manderla. The truss and loading for the four examples are precisely the same.

In order to compare the results some computations were made so that the stresses will be given for the same fibers. In the graphic method of deformation contour the stress is found for the fiber where the moment and the stress have the same sign. In the analytic method the stress is found for the fiber which is first met with in passing around the end of the member in a clockwise direction. These two conventions are identical in result but are different from those adopted in the above mentioned articles. For members having symmetrical sections this difference of conventions makes a difference in sign but for members having unsymmetrical sections, it also makes a difference in magnitude. To facilitate comparison only one convention should be used in the table—the one adopted agrees with this paper.

The top chord members of the truss analyzed in the above examples have an unsymmetrical section and the values of y are different for the top and bottom fibers, being 8.8″ for the top and 11.6″ for the bottom. Since the examples worked out in this paper used a value of y that is for the top fiber of the chord (equal to 8.8″), the fiber stress is not correct if it applies to the bottom fiber. The ratio of (y for bottom fiber) to (y for top fiber)—equal to $11.6/8.8 = 1.32$—should therefore be applied, in order to secure the correct results.

TABLE OF COMPARISON

Member	End	Fiber	Secondary Stresses in 1000 lbs./ in.^{2} by			
			Mohr's Method	Manderla's Method	Graphic Method	Analytic Method
1–2	1	T	–0.12	–0.12	–0.12	–0.12
	2	B	–0.01	–0.04	–0.03	–0.03
2–4	2	T	0.11	0.11	0.09	0.11
	4	B	0.43	0.45	0.44	0.45
4–6	4	T	–0.33	–0.33	–0.33	–0.33
	6	B	0.13	0.13	0.13	0.12
1–3	1	T	0.29	0.29	0.29	0.29
	3	B	0.36	0.35	0.34	0.35
3–5	3	T	–0.29	–0.28	–0.29	–0.28
	5	B	0.08	0.10	0.10	0.09
5–7	5	T	0.02	0.03	0.03	0.03
	7	B	0.41	0.42	0.42	0.39
2–5	2	T	–0.16	–0.17	–0.18	–0.16
	5	B	0.01	0.01	0	0
5–6	5	T	–0.17	–0.16	–0.16	–0.17
	6	B	0.06	0.06	0.06	0.06
2–3	2	R	–0.41	–0.44	–0.44	–0.44
	3	L	–0.46	–0.50	–0.50	–0.51
4–5	4	R	–0.36	–0.34	–0.34	–0.33
	5	L	–0.36	–0.31	–0.31	–0.31
6–7	6	R	0	0	0	0
	7	L	0	0	0	0

XVI. SECONDARY STRESSES IN SPECIAL FORMS OF BRIDGE TRUSSES

Bridge trusses of unusual types require special treatment in the analysis of secondary stresses. Each is a problem of itself and can only be solved by judicious changes in the procedure. A thorough understanding of the principles involved is very necessary as no hard and fast rules can be laid for all cases. Suffice it to say that as long as the truss is composed of triangular elements and there are at least two joints where only two members meet, the solution can always be

effected by the methods presented in this paper (see Section XIV), although in certain cases the graphical method may not be as expedient as for simple trusses.

To illustrate the wide application of the methods given in this paper let a Baltimore truss as shown in Fig. 22 be analyzed for secondary stresses by the analytic method with scientific arrangement of computations.

As for simple trusses the first step of procedure is to find the change of angles due to the action of primary stresses. For a truss with subdivided panels, as that shown in Fig. 22, difficulties are encountered in applying equation (2) to a figure like 4698, as the members 46 and 69 are no longer on a straight line when the bridge is under load. In such a case* assume a member inserted within the four-sided figure so that it will divide the space into two triangles, as member 68 shown by dotted lines in the figure. The deformation of this imaginary member can be calculated by finding the relative deflection of the two joints which it connects, assuming the sectional area of the inserted member to be zero. From the deformation thus obtained the changes in angles of figure 4698 may be determined in the usual way by using equation (2). As this equation contains unit stresses in the three sides of the triangle it will be necessary to find the unit stress in the imaginary member due to its deformation. For a panel as shown in Fig. 23 *a* or Fig. 23 *b*, the unit stress (or strain) in the imaginary member is†

$$P_8=2(P_1+P_2)\frac{d^2}{c^2}+2(P_3+P_4)\frac{h^2}{c^2}-(P_5+P_6+P_7).$$

By the use of this equation the changes in angles in panel 59 of Fig. 22 may be found by substituting the values of P in equation (2). After these values are known the K-tables in the analytic method may be filled with proper values as described in Section XIII.

In the solution of the problem there is no difficulty in the process until member 45 is reached. The values of f_{45}, f_{54}, r_{45}, and r_{54} are

*See Johnson, Bryan and Turneaure's Modern Framed Structures, Part II, p. 476.
†See Johnson, Bryan and Turneaure's Modern Framed Structures, Part II, p. 477.

all expressed in terms of x and y, the secondary stresses in member 12 at ends 1 and 2 respectively. To find the stresses in member 56 it is necessary to know r_{56} and r_{65}. The value of r_{56} is obtained by passing a deformation contour, analytically, around joint 2 with the known value of r_{53}, but to find r_{65} from r_{56} it is necessary to know f_{56}. Now, around joint 5 the stresses f are known only for three members—f_{53}, f_{52} and f_{54}—while there are five members entering the joint. The value of f_{56} cannot, therefore, be found from an equilibrium equation. To overcome this difficulty assume the value of r_{65} be known. If z be used to denote this value, then $z=a\,x+b\,y+c$, where a, b, and c are constants. From the assumed value of r_{65} and r_{56} the values of f_{56} and f_{65} are obtained. An equilibrium equation around joint 5 then determines the value of f_{57} in terms of x, y and z. The value of r_{57} being known from r_{53}, the value of r_{75} may be determined from f_{57}. With this modulus, r_{75}, a deformation contour may be passed around joint 7, giving r_{76}, and r_{79}—all expressed in terms of x, y and z. A deformation contour passed around joint 6 with r_{65} equal to z furnishes the values of r_{64}, r_{67} and r_{69}. From r_{67} and r_{76} the values of f_{67} and f_{76} are obtained. An equilibrium equation around joint 7 then determines f_{79}. This value of f_{79} together with that of r_{79} gives r_{97}. From r_{97}, r_{96} is obtained, and from r_{96} and r_{69}, f_{69} is obtained. Around joint 4, r_{46} is known from r_{42} and when combined with r_{64} from the assumed r_{65} the stresses f_{46} and f_{64} are determined. Up to this point all the values in panel 59 are expressed in terms of x, y and z and all stresses around joint 6 being now known it is possible to find the value of z by an equilibrium equation. That is,

$$S_{65}\,f_{65}+S_{67}\,f_{67}+S_{69}\,f_{69}+S_{64}\,f_{64}=0.$$

This will give the value of z in terms of x and y. By substituting this value of z, all the values of f in panel 59 are then expressed in terms of x and y. This may be done by writing the values of z (in terms of x and y) on a strip of paper and slide this strip under the values of f for addition or subtraction.

The stresses in members 48 and 89 are obtained in the usual way. The imaginary member 68 is not considered any more after the values of K are known.

To arrange the computations for this truss it is necessary to make some slight changes in the form as shown in Fig. 24. Being familiarized with the method of constructing Plate IV a detailed explanation for this change is not necessary.

XVII. FEATURES OF THE NEW METHODS

1. Features common to Both Methods

(1) No simultaneous equations involving more than two unknowns are required.

(2) Both methods handle a quantity which is of the same dimensional degree as the secondary stress itself. This gives more accurate results than when using a quantity involving large multiples of f.

(3) Both methods take much less time than the ordinary methods.

(4) There are a number of checks available for each method, especially for the graphic method.

(5) Both methods can be easily applied to influence lines of secondary stresses.

(6) In both methods the complete information is contained on only one sheet of paper, including the rough computations, figuring, data, and formula.

2. Features of the Graphic Method of Deformation Contour

(1) It is strictly graphical. From beginning to end no computations are requird.

(2) It gives a complete representation of secondary stresses in every member of the truss.

(3) The method not only shows secondary stresses at the two ends of a member, but also the variation of the stress along its whole length.

(4) It gives the point of inflexion in a member and also its form of bending.

(5) With the aid of the force diagram for secondary stress lines this method shows how the secondary stresses are affected by changing the section moduli of the members.

(6) It is more accurate than the ordinary analytic methods if the latter be computed by the use of a slide rule. This is due to the fact that in analytic expressions the secondary stress is often found from the difference of two large numbers which must be very accurate, since the difference is small.

(7) The errors made in locating the base lines and vertex lines are not cumulative, as the effect of one line upon the other is very small.

(8) It has a sufficient number of checks at every step of procedure.

3. Features of the Analytic Method with Scientific Arrangement of Computations

(1) In bridge offices where a large number of standardized trusses are to be analyzed for secondary stresses, the truss diagram together with the small tables could be blue printed.

(2) This method is semi-graphical in that it shows the stresses at the proper places and the effects of one upon the other.

(3) The entire process is mechanical. Every figure has a definite space; every procedure has a definite order.

(4) No figure is to be recorded twice and no necessary figures are omitted.

(5) All rough computations and figuring are shown on the diagram thus rendering possible a check at any future time.

(6) The figures on the diagram are so arranged that all the additions and subtractions do not involve more than two lines of figures and can be accomplished without the use of extra paper, as when three columns of figures are to be added, etc. All the other computations can be made on a slide rule.

(7) The method is capable of affording any degree of accuracy which may be desired.

(8) Practically all back references are eliminated.

(9) It has a unique check of secondary stresses as soon as they are found.

SECONDARY STRESS: A FACTOR IN DESIGN

XVIII. INTRODUCTION

While the importance of secondary stresses is still a moot question, it cannot be denied that great improvements have been made in the design of structures as a result of the studies and observations made on the subject during recent years. The achievement so far attained is very beneficial and convincing although it is not uncommon to hear it said that secondary stresses accompany good design, as it makes the structure rigid and stiff which is highly desirable. To be sure, a good design does aim at a rigid structure but certainly not at the expense of its strength. Structures may be of different types: Some are rigid, others are strong, and still others are both rigid and strong. A bridge may be designed and detailed in different ways: Some make the structure rigid, others make it strong, and still others make it both rigid and strong. Would it be logical to consider only rigidity in a structure—in the selection of types, proportioning of members, and designing of details—and neglect its effects on the most vital factor in the design—the strength?

The consideration of secondary stresses in a design tends to make the structure both rigid and strong, not only in type and design but also in details. Eventually the design is also economical as the uncertain "factors of safety" may be greatly reduced. It is for this reason that the secondary stress is so important in design and has been so widely considered. It must be admitted, however, that the subject is a difficult one—not in the theoretical analysis of course, but in its practical application. Oftentimes a design made to satisfy secondary stresses violates other good principles which must be respected. In other cases, the gain in reduced secondary stresses does not balance the loss of rigidity. Here good judgement must be exercised to determine which course to pursue.

Before going into the details of design some general conclusions about the secondary stresses will be given.

(1) The secondary stresses are, in general, proportional to the primary stresses, and, therefore, are conveniently expressed in percentages of primary stresses.

(2) Other things being equal, the percentages of secondary stresses are proportional to the distance from the gravity axis to the outer fiber in the plane of bending, and inversely proportional to the length of the member.

(3) The secondary stressses in any member depend on the distortion of all the members of the truss, but primarily upon the distortion of members of the triangles of which this member is a part.

(4) A design in any individual member cannot be changed without affecting the secondary stresses in the other members.

In the following sections are given some principles of design which must be observed in reducing secondary stresses. These are largely derived from theoretical studies and practical observations and are very valuable on that account. During the past few years as increasing attention was paid to secondary stresses, considerable space in books and periodicals has been devoted to the design of structures from this point of view. This information, however, is not within easy reach as it is scattered in a vast amount of literature and the subject has never been systematically treated. As one of the purposes of the present paper is to present the facts about secondary stresses which must be understood by the designer, these principles of design will be listed under proper headings and discussed and digested in a logical order. For each principle stated the source of information is accredited by using a letter (explained in the following key) to indicate the title of the work. This is followed by a number referring to the page.

Key

a 4 = Proceedings of the American Railway Engineering Ass'n, 1914, p. 438.

a 6 = Proceedings of the American Railway Engineering Ass'n, 1916, p. 129.

e = Engineering News Record.

g = Grimm's Secondary Stresses in Bridge Trusses.
j = Merriman & Jacoby's Roofs and Bridges, Part III.
k = Kunz's Design of Steel Bridges.
M = Thesis by T. Mao presented to Cornell University.
m = Molitor's Kinetic Theory of Engineering Structures.
p = Secondary Stresses in Framed Structures by Pitman in Proceedings of Engineers' Society of Western Penn., Vol. 25.
r = Notes on Design by Reichman in Journal of Western Society of Engineers, Vol. 17.
s = Stress Measurement on the Hell Gate Arch Bridge by Steinman in Transactions of the American Society of Civil Engineers, Vol. 82.
TII = Modern Framed Structures, Part II.
TIII = „ „ „ Part III.
t = Thayer's Structural Design.
W = Waddell's Bridge Engineering.
w = Effects of Secondary Stresses on Design by Wilson in the Journal of Western Society of Engineers, Vol. 21.

XIX. SELECTION OF TRUSSES

(1) In choosing between different styles of trusses, those of the statically determinate class should always receive preference, other things being equal. The primary stresses will usually be less than in similar indeterminate systems, especially when the temperature stresses are included. Yet the deformation and secondary stresses may be less, and frequently the connections may be simpler for the indeterminate type. (m 268)

(2) Among the statically indeterminate structures the two-hinged arch is especially immune from secondary stresses. (s 1840, M 497)

(3) The secondary stresses in continuous trusses are very large at the centre and in its neighborhood and for this reason pins at these points appear to be advisable in order to reduce the stresses. (g 129)

(4) The best modern practice in bridge engineering does not countenance the building of trusses having more than a single system of cancellation. (W 271). The secondary stresses in such a system are

generally very high, often reaching 100 per cent of the primary (TII 490), as the distortion in truss members due to loads on one system only is very great. (t 235, g 129).

(5) In double triangular trusses the secondary stresses can be greatly reduced by the insertion of verticals connecting every pair of upper and lower joints. These verticals effectively connect the two single systems. (TII 491, g 123).

(6) Trusses consisting of approximately equilateral triangles, and without hangers or vertical struts, present the most uniform condition and will have, in general, the lowest secondary stresses. (TIII 93, a4).

(7) A truss composed of right-angled triangles will show somewhat higher secondary stresses, and such stresses will be large if the ratio of height to panel length is large. (TIII 94, a4).

(8) The truss systems should be as simple as possible and all members which make the stress distribution uncertain should be avoided. (k 169).

(9) As far as the elimination of secondary stress is concerned, the Pratt truss, the Warren truss with verticals, and the K-truss are very desirable. (W 200).

(10) The amount of secondary stress in ordinary Pratt trusses varies from 15 to 35 per cent of the primary stress. (a6).

(11) The secondary stresses in the Warren system are often less than those in the Pratt system. (TII 487).

(12) In single Warren trusses without verticals the secondary stresses in chords increase from the center of the span toward the end, while for the web members these stresses increase in the opposite direction. This circumstance is fortunate, as the web members in the region of the center of the span often have an excess in section and the end members of both the top and the bottom chords show this surplus still oftener. (g 105).

(13) In a riveted truss, the span should be made as deep as good designing will permit, for two reasons: (1) The connections of the diagonals to the chords may be made smaller, (2) the deformation of the truss under load may be minimized. Both of these effects tend to decrease the secondary stresses. (r 95, TII 492).

(14) In trusses with subdivided panels, the secondary members cause great secondary stresses in the main members of the truss on account of their distortion. (TII 492, t 235, W 201).

(15) The best arrangement, so far as the secondary stress is concerned, is where each web member forms an integral part of the entire truss so that its stress will gradually change as the load progresses.

(16) In a Baltimore truss, the use of a sub-strut is much more advisable than the use of a sub-tie. (TII 494).

(17) The use of collision struts cannot be recommended, because they divide the end posts into two fragments, decreasing the ratio between length and width of these members, pulling the posts out of line and increasing the secondary stresses. The collision struts are, as a rule, rather weak members and it is a question whether it would not be better to use the material on the end posts instead of on these struts, thereby increasing their strength in the direction of the plane of the truss as well as at right angles to their plane. (g 130, TII 458).

(18) The double triangular truss with sub-panels is satisfactory so far as the action of the secondary stress is concerned. (TII 495).

(19) A truss with polygonal chord as the parabolic truss, or the Schwedler truss, is a good selection. (g 129).

(20) In a truss, aim to make the curve of deflection an approximate circle with vertical members radial. This means uniform stress in chords and low stresses elsewhere. (t 235).

(21) Curved members, in which the neutral axis is not straight before the application of the load, should never be tolerated under any circumstance. (W 272, g 129, k 170).

XX. ARRANGEMENT OF SPAN

(22) Skew bridges and bridges on curves should never be built except in very rare cases where no other disposition is possible. These types should be regarded as measures of last resort. (m 267, W 271).

(23) Wherever possible the loads should be applied at the panel points only. (k 170).

(24) The plane of the lateral system should coincide with that of the chords; otherwise, the stresses in webs and chords will produce bending moments in the posts. (r 102, m 268, k 185).

(25) The plane of the floor should be as close as possible to that of the lateral system. (m 268).

(26) The gravity axes of all the main members of trusses and lateral systems coming together at any apex of a truss should intersect at a common point wherever such an arrangement is practicable. In other words, the panel points of the bracing should coincide with the panel points of the main trusses. (W 273, k 185, g I30).

(27) If horizontal trusses are used in the floor system to resist traction stresses, these should be placed near the center of a span or midway between expansion joints. (TIII 105, a6).

(28) The double intersection Warren truss with verticals is very well adapted to lateral bracing. (TII 492).

(29) Brackets on posts should be avoided wherever possible. (g 130).

(30) The pedestal pins in riveted trusses should not be placed below the bottom chords but on the center-lines. This eliminates the bending moments that may be developed in the end posts and the end panels of the bottom chords by the train thrust. (W 284).

XXI. DESIGN OF FLOOR SYSTEM

(31) The floor system should be so designed that it is rigid in itself with respect to its main duty, but independent, so far as practicable, of the action of the chord system of the main trusses. (TII 505).

(32) Floor-beams should be made deep to reduce secondary stresses in the cross frames. (m 268, r 101).

(33) The flanges of the floor-beams should be made relatively narrow. (TII 505).

(34) The floor-beams should be centrally connected to the post and not too far below the plane of chords. (r 101).

(35) Flexible connections are advisable between floor-beams and truss members. These should be so arranged that the end connections are rigid in a vertical plane but flexible in a horizontal plane. (m 268, g 130).

(36) Floor-beams should be provided with brackets or any other suitable construction to prevent the bending of the horizontal flanges. (g 132).

(37) The stringers should be made heavy and continuous and should be designed to transmit the tractive forces to the panel points of the loaded chord instead of to the floor-beams by inserting proper tie members between the stringers. (W 400, m 268).

(38) When the floor-beams extend considerably below the bottom of the stringers they should have stiffeners at or near the stringer connection to transmit to the flanges of the floor-beam the bending caused by the deflection of the stringers.

(39) In long-span bridges, stringers should be provided with expansion joints at intervals of a few panels. (TII 505, r 101).

(40) The connection of lower laterals to stringers is of doubtful value. Owing to the relative movement between the chords and the floor system, such a connection would cause considerable lateral bending in the stringers. The laterals are therefore better merely supported on the stringers. (TIII 104, a6).

XXII. PROPORTIONING OF MEMBERS

(41) The more uniform the proportions of a truss, the less, in general, will be the secondary stresses. Sudden changes in length, in width, or in moment of inertia, are likely to result in relatively large secondary stresses. (TIII 93).

(42) A reduction of moment of inertia of any member tends to increase the deflection of that member and reduce that of others. To maintain a balance in the fiber stresses themselves, it is necessary to make the width of a member correspond in a measure with its moment of inertia. Wide members of small moment of inertia are likely to have high secondary stresses; narrow and compact members will have

low secondary stresses. This statement does not take account of the long column action in compression members, the effects of which tend in the opposite directon. (TII 487).

(43) It is advisable to keep the moments of inertia of web members as small as possible. (t 235).

(44) The axes of all members of a truss should be in the same plane and should intersect at a common point at all connections. (m 267, W 273).

(45) In any truss, no torsion on any member should be allowed if it can possibly be avoided; otherwise, the greatest care must be taken to provide ample strength and rigidity for every portion of the structure affected by such torsion. (W 272).

(46) If the axes of members do not intersect at one point, the secondary stresses developed due to eccentric connections may be as high as 1 to 1 1/2 times the primary stresses. (p).

(47) Excessively deep members should be avoided (W 495), especially in tension chords and in chords of trusses with sub-divided panels (a6). However, the members should not be so slender as to impair economy of design (k 177) or to reduce the effectiveness of the riveted connections. (r 95).

(48) To secure a comparatively narrow width, and at the same time preserve the stiffness against buckling, a compression member may be tapered from the center toward both ends. (m 267).

(49) To avoid excessive bending stresses in posts to which floor-beams are riveted, the posts should be made only of moderate width in a transverse direction. In the best design these bending stresses are likely to be as much as 25 per cent of the primary stresses. (TIII 105, g 129).

(50) If the floor-beams connected to the posts are shallow, the latter should be given increased sectional areas to provide for the secondary stresses due to the deflection of the floor-beams. (j 401).

(51) Truss members and portions of truss members should always be arranged in pairs symmetrical about the plane of the truss (W 273). This will tend to equalize the stresses carried by the component parts of the member.

(52) The cross-sections of members should be so chosen that the material is concentrated as far from the neutral axis as possible, thus securing the largest moments of inertia for the smallest over all dimensions. (g 129). The cross form is thus the least advantageous while a square box form is the most desirable. However, consideration must be given to the fact that when the secondary stresses occur simultaneously in the plane of the truss and in a cross frame, the stresses are additive in members of the box type but not in those with cross forms. (m 267).

(53) Since the effect of the hip verticals and suspenders in a truss is to pull the chord members out of line, they should be provided with liberal cross-sections. (TII 487, a6).

(54) Where the suspenders are of considerable length and attached to the lower chord, it is desirable to make them slightly shorter than the calculated length. (a6).

(55) Where a top chord in a subdivided truss is supported by means of secondary members, these should be made slightly longer than the calculated length. (a6).

(56) The secondary stresses in chord members may be reduced by the proper selection of cross-sections for the diagonals and their attachments to the chords. (r 101).

(57) In case a member is made up of a single part, like an angle, the gage lines for rivets should be as close to the gravity axis as possible. (p).

(58) For a truss with built-up, continuous chords and built-up diagonals, pin-connected throughout, the secondary stresses will be very low if the chords are made sufficiently shallow and the diagonals sufficiently wide in the plane of the truss, so as to overcome the pin friction. (r 96).

(59) In eye-bar trusses, the eye-bars should be made as wide as permissible in the plane of the truss. (r 97).

XXIII. DESIGNING OF DETAILS

(60) Riveted connections must be made concentric by so grouping the rivets that they will balance about center-lines and center planes to as great an extent as possible. (W 274, g 129).

(61) The end lateral connections should be as concentric as practicable. This is very important in the case of the end panel of the lower chord, as eccentric connections at this point result in heavy secondary stresses in end posts and lower chords. (TIII 105).

(62) The use of thick gusset plates and large diameter rivets is advisable as this practice will materially reduce the number of rivets and the size of the plates. (m 267, W 491).

(63) In case two gusset plates are used at each joint these should be firmly connected together by the use of diaphragm plates.

(64) In a pin-connected truss, the diameters of the pins should not be so large that the friction developed around the joints will virtually make the connections rigid. They should therefore be made as small as is consistent with the design. (m 268, g 132).

(65) If the diameter of the pin be made three quarters the width of the eye-bar, as is usually done, the secondary stresses developed will be about 45 per cent of the primary stresses with the coefficient of friction equal to 0. 2. (g 77).

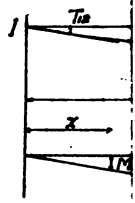
(66) A double pin arrangement has been evolved for bettering the pin bearings in cantilever spans, whereby a second pin is placed side by side with the bearing in the bottom chord. The object is to relieve the bending stresses in the bottom chord, which would result from the simultaneous deflections of the two adjacent spans. (W 1067).

(67) For lacing of compression members, the arrangement of double lacing with transverse cross-bars is advisable. (TII 498).

(68) Long diagonals which are subject to the bending due to deflection of floor-beams should preferably be provided with lacings. (j 401).

(69) For heavy members composed of two parts, one on each side of the centre-line, the use of cross diaphragms spaced at 6 to 10 feet is necessary. (W 506). This not only holds the members true to shape and line but also tends to equalize the stresses in the two component parts.

(70) The use of lug angles for connections is advisable for members having single angle sections.



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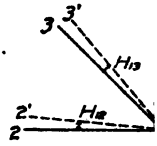

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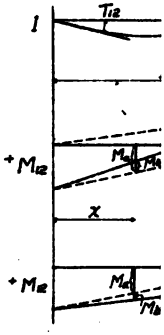

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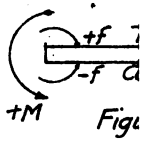

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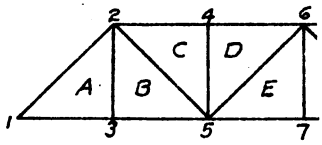

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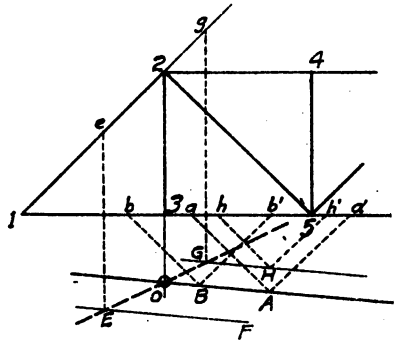

Figure 15

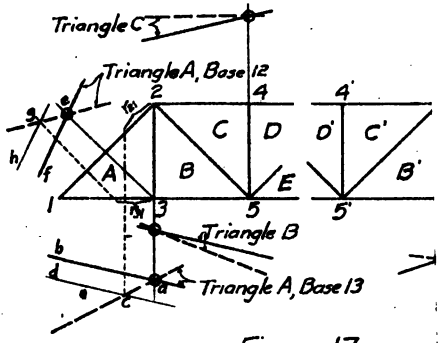

Figure 17

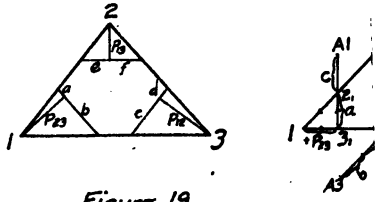

Figure 19

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ERRATA

Page	Line	
2	27	Comma after "for illustration (Fig. 1)"
3	13	"Let 12, 13, 14, and 15 be four members" should read "Let 12, 13, 14, 15 and 16 be five members"
4	6	"$H_{12} - H_{13}$" should read "$H_{13} - H_{12}$"
4	17	"$H_{12} - H_{13}$" should read "$H_{13} - H_{12}$", "$H_{13} - H_{14}$" should read "$H_{14} - H_{13}$"
5	7	This paragraph should be revised to read as follows: "It will be seen in equation (d) that the first two terms of the right hand member represent the work done on ends 1 and 2 of beam 12 by M_{12} and M_{21} respectively due to angular rotations caused by the respective moments—by equation (a)—and the last term is the sum of the work done by M_{12} at 1 and M_{21} at 2 due to the angular rotation caused by the moment at the other end. If δ_{12} is the angular rotation at 1 caused by a unit moment at 2 and δ_{21} is the angular rotation at 2 caused by a unit moment at 1, the total angular rotation at 1 due to M_{21} at 2 will be $M_{21}\,\delta_{12}$ and that at 2 due to M_{12} at 1 will be $M_{12}\,\delta_{21}$. The work done by M_{12} at 1 due to the action of M_{21} at 2 is therefore $\frac{1}{2}M_{12}\,M_{21}\,\delta_{12}$ and that by M_{21} at 2 due to the action of M_{12} at 1 is $\frac{1}{2}M_{21}\,M_{12}\,\delta_{21}$. But by Maxwell's theorem of reciprocal deflections, $\delta_{12} = \delta_{21}$, each being equal to $L/6EI$, and hence these two kinds of work done are equal. The difference of W_{12} and W_{21} is therefore simply the difference of the first two terms in the right hand side of the above equation and $$W_{12} - W_{21} = \frac{1}{2}\frac{1}{EI}\left(\frac{M_{12}^2 L}{3} - \frac{M_{21}^2 L}{3}\right) \qquad (e)$$"
13	6	"if sides," should read "if the sides,"
13	12	"K_{plm}/U_{lm}" should read "$-K_{plm}/U_{lm}$"
14	10	"joint 1 in this case" should read joint l in this case"
14	23	"sides 53 and 52" should read "sides 52 and 53."

PAGE	LINE	
23	8	"r_{12}" should read "r_{13}"
24	9	"as r_{23}, r_{32} and r_{21} . . " should read "as r_{23}, r_{32} and r_{31} . . "
26	26	"$r_{/2/}$" should read "$r_{1/2/}$"
35	20	"for angles" should read "for the angles"
40	16	"forth" should read "fourth"
40	33	"—2.56" should read "—2.46"
42	20	"0—1.33=—1.33" should read "0—.133=—.133"
42	34	"—2.403" should read "—2.405"
44	14	"above k" should read "below k"
45	17	This equation should be designated as (18)
46	4	"fifth and sixth columns" should read "sixth and seventh columns"
46	6	"third column" should read "fourth column"
46	7	"fourth column" should read "fifth column"
49	30	Insert after "the values of z" "multiplied by its coefficients"
50	22	"requird" should read "required"

Plate II Fig. 20. Letters "3" at the right-angled vertex and "3_2" between segments b and d omitted in the figure.

NOTE 1. The term "contour" used in this paper has a meaning different from that used in topographic surveying.

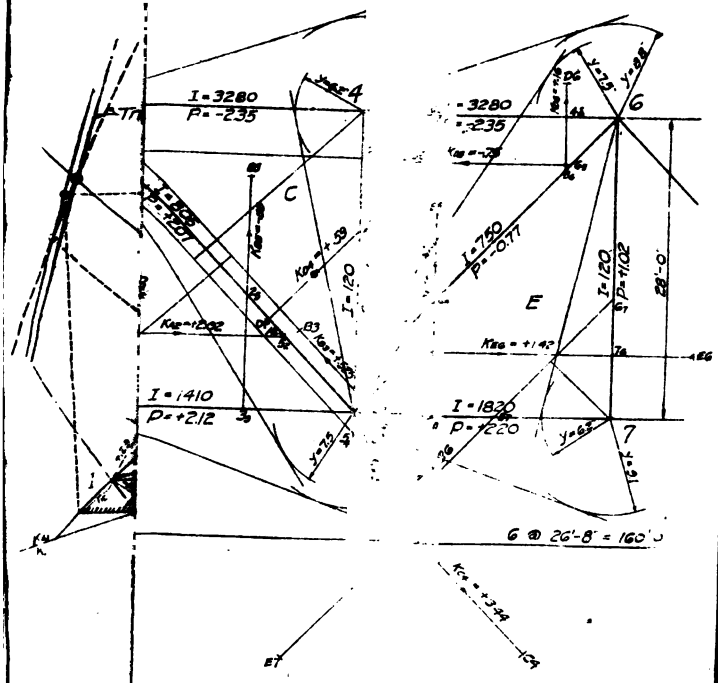
NOTE 2. In equation (2), page 4, the sign of δA agrees with the convention in this paper if for P. plus sign is used to denote tension and minus to denote compression.

NOTE 3. In Section III, equations (7), (8), (11) and (12) have no direct bearing upon the new methods developed in this paper and should have been omitted for simplicity.

NOTE 4. Before reading Section XI, on General Procedures, it is advisable to take up the first two paragraphs of Section XIV, on the Construction of Influence Lines. This will give a better understanding of the analytic method.

TANGSHAN, CHINA, T. E. M.

March, 1921.



...ONDARY STRESSES

By

THE METHOD OF

...FORMATION CONTOUR

August, 1919

T.E.M.

PLATE III

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FORMULA:

$K_{nm} = (P_{mq} - P_{mn}) \cdot \cot \beta/2$

$K_{nq} = (P_{mn} - P_{mq}) \cdot \cot \beta/2$

$f_{mn} = a_{mn} x + b_{mn} y +$
a, b & k are Co

$r_{mn} = 2 f_{mn} - f_{nm}$
$= \frac{U_{mn}}{U_{nn}} r_{mp} + \frac{\Sigma_{mn}}{\Sigma_{nn}}$ f
the Reference M
and ΣK is Nega
Joints

$r_{nm} = 3 f_{mn} - 2 r_{mn}$

$f_{mn} = \frac{1}{3} r_{mn} + \frac{1}{3} (r_{mn} + r_{nm}$

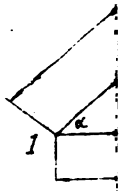
$f_{mn} - f_{nm} = \frac{1}{3} r_{nm} - \frac{1}{3} r_{mn}$

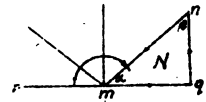
$M_m = \Sigma S_{mn} f_{mn} = 0$

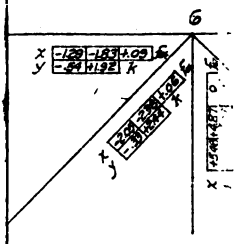
$f_{67} - f_{76} = 0$

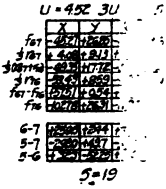
Check for Every Triang

$U_{mq} (f_{qm} - f_{mp}) + U_{mq} (\quad - f_{mq}) = 0$









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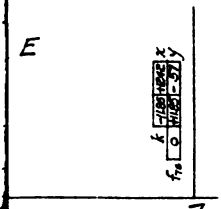
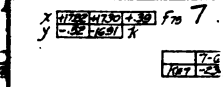


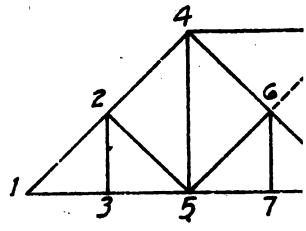


...NDARY STRESSES

By

...NALYTIC METHOD

August, 1919 T.E.M



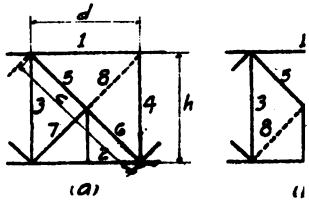
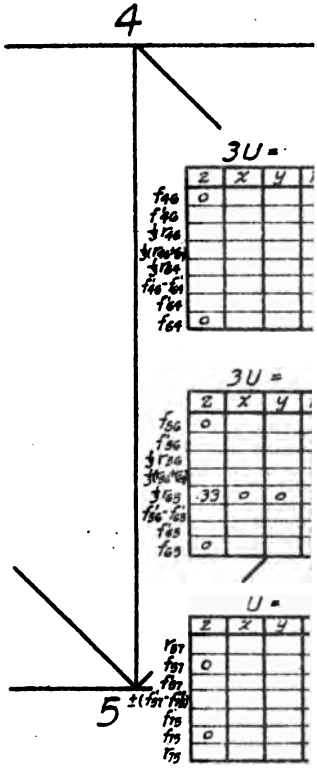



Figure 23